

Modeling and Synchronization of Chaotic Gyroscopes using TS Fuzzy Approach

Malihe Sargolzaei^{*1}, Mahdi Yaghoobi² and Rajab Asgharian Ghannad Yazdi³

^{1,2} *Department of Electrical Engineering, Mashhad Branch,
Islamic Azad University, Mashhad, Iran.*

³ *Department of Electrical Engineering, Ferdowsi University of Mashhad, Iran.*

Abstract

Gyroscope is important component in guided navigation systems, aerospace and robotics. Uncertainty in nonlinear dynamics model of gyroscope system and increasing of time needed to control of motion of gyroscope can lead to large losses. In order to reducing of uncertainty and linearization of gyroscope system, Takagi–Sugeno (TS) fuzzy model was used. TS fuzzy controller was applied to synchronization of two chaotic gyroscope systems. We have shown that synchronization of two gyroscopes is done in short time.

Keywords: Chaos, Takagi–Sugeno fuzzy model, Indirect model reference, Nonlinear system, Gyroscope.

1. Introduction

Gyroscopes as one of the most important elements used in aerospace, marine systems, robotics and optics industries widely. This system has been studied by many authors [1-8]. In the past decade, a new phenomenon in this system was expressed and many researchers were interested in this phenomenon. This phenomenon was first described in 1981 and it has been shown that the system has a chaotic behavior for certain initial conditions [9]. Gyroscopes are used in sensitive industries and the slightest error can cause a lot of damage. Therefore, an appropriate control signal must be applied to the system and the chaotic behavior will be controlled. Since the studied system has large uncertainty in this paper, we have proposed a fuzzy controller. Because it is used in online applications, so it should be tried to reduce the time control. For this purpose, we are changing the gyroscope nonlinear model to the TSk fuzzy model. Within this

method, a nonlinear system is transformed into linear or pseudo-linear subsystems [10].

In this paper, controllers are designed for the obtained model. The objective in this paper is synchronization of two chaotic gyroscopes. This paper is organized as follows: Gyroscope equations and chaotic conditions are described in Section 2. In section 3 the TS fuzzy model is given. In section 4 the controller is designed to synchronize two chaotic systems. Numerical simulations in Section 5 and conclusions in Section 6 are given.

2. Motion equation of a gyroscope

A symmetric gyro with linear-plus-cubic damping is considered [1]. These equations are given with respect to the angle theta:

$$\ddot{\theta} + \alpha^2 \frac{(1-\cos \theta)^2}{\sin^3 \theta} - \beta \sin \theta + c_1 \dot{\theta} + c_2 \dot{\theta}^3 = f \sin \omega t \sin \theta \quad (1)$$

where $f \sin \omega t$ is a parametric excitation, $c_1 \dot{\theta}$ and $c_2 \dot{\theta}^3$ are linear and nonlinear damping terms, respectively. the $\alpha^2 \frac{(1-\cos \theta)^2}{\sin^3 \theta} - \beta \sin \theta$ is a nonlinear resilience force. Considering the states as $x_1 = \theta, x_2 = \dot{\theta}$ and $g(\theta) = \alpha^2 \frac{(1-\cos \theta)^2}{\sin^3 \theta} - \sin \theta$, the equations are normalized as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\alpha^2 \frac{(1-\cos x_1)^2}{\sin^3 x_1} - c_1 x_2 - c_2 x_2^3 + \beta \sin x_1 + f \sin \omega t \sin x_1 \end{cases} \quad (2)$$

In particular, the gyro exhibits chaotic behavior for the following parameters: $\alpha^2 = 100, \beta = 1, c_1 = 0.5, c_2 = 0.05, \omega = 2, f = 35.5$ and $[x_1(0), x_2(0)] = [1, -1]$.

3. Takagi-Sugeno fuzzy model of nonlinear chaotic gyro

In order to achieved a control model, in this system we have briefly made a review on TS fuzzy model [11]. This control oriented fuzzy modeling method is simple and natural. The system dynamics is captured by a set of fuzzy implications which characterize local relations in the state space. The chaotic system considered is in the form of $\dot{x}^n = f(x) + g(x)u + d(x, t)$, in which $f(x)$ and $g(x)$ are unknown smooth functions and $d(x, t)$ is the bounded external disturbance. This nonlinear chaotic system can be modeled using a Takagi–Sugeno (TS) fuzzy model. The main feature of a TS fuzzy model is to express a nonlinear dynamical system by a set of fuzzy rules. Each fuzzy rule is a linear dynamical system and the overall fuzzy model of the system

is achieved by a fuzzy blending of each local model. The i th rule of the fuzzy model for the nonlinear system can be written as:

$$R^i : \text{if } x_1 \text{ is } M_1^i(x_1) \text{ and } \dots \text{ and } x_n \text{ is } M_n^i(x_n) \text{ then } \dot{x} = A_i x(t) + B_i u(t) + B_1 d(x, t) \quad (3)$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)] \in R^n$ denotes the state vector and $i = 1, \dots, l$; $u(t) \in R$ is the input; l is the number of rules. Also A_i, B_i and B_1 are state matrices.

The final output of the fuzzy model is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^l h_i(x(t)) \{A_i x(t) + B_i u(t)\} + B_1 d(x, t) \quad (4)$$

which $h_i(x(t)) = w_i(x(t)) / \sum_{i=1}^l w_i(x(t))$. It is the firing strength of i th rule, and

$$w_i(x(t)) = \prod_{j=1}^n M_j^i(x_j(t)), \quad (5)$$

where $M_j^i(x(t))$ is the grade of membership function.

Using TS fuzzy model, the gyroscope nonlinear dynamic model becomes the three linear subsystems. Given the stated assumptions, the fuzzy model of system as follows:

$$\begin{aligned} &\text{if } x_1 \text{ is zero then } \dot{x}_2(t) = Z_1, \\ &\text{if } x_1 \text{ is positive then } \dot{x}_2(t) = Z_2, \\ &\text{if } x_1 \text{ is negative then } \dot{x}_2(t) = Z_3, \end{aligned} \quad (5)$$

where

$$\begin{aligned} Z_1 &= 35.5 \sin(2t - 24) (x_1 - 0.001) - 0.5x_2 \\ &\quad + 0.0355 \sin(2t) - 0.024 \\ Z_2 &= 31.1542 \sin(2t - 30.9453) (x_1 - 0.5) \\ &\quad - 0.5x_2 + 17.0196 \sin(2t) - 13.1201 \\ Z_3 &= 31.1542 \sin(2t - 30.9453) (x_1 + 0.5) \\ &\quad - 0.5x_2 - 17.0196 \sin(2t) + 13.1201. \end{aligned}$$

4. Synchronization of two chaotic gyroscopes

Consider two coupled, chaotic gyro systems are as following:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\alpha^2 \frac{(1 - \cos x_1)^2}{\sin^3 x_1} - c_1 x_2 - c_2 x^3 + \beta \sin x_1 + f \sin \omega t \sin x_1, \end{cases} \quad (6)$$

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = -\alpha^2 \frac{(1-\cos y_1)^2}{\sin^3 y_1} - c_1 y_2 - c_2 y^3 + \beta \sin y_1 + \\ f \sin \omega t \sin y_1 + U(t) + \Delta f(y_1, y_2), \end{cases} \quad (7)$$

where $U(t)$ is the control input, $\Delta f(y_1, y_2)$ is an uncertainty term representing the unmodeled dynamics or structural variation of the system which is given in Eq. (7). In general, the uncertainty is assumed to be bounded as $|\Delta f(y)| \leq \alpha$, where α is a positive constant. Control input is applied to a slave system (7) that will be synchronized with the master system. Therefore the following objective must be achieved:

$$\lim_{t \rightarrow \infty} \|x(t) - y(t)\| \rightarrow 0, \quad (8)$$

Because the three linear subsystems for gyro is intended, with definition of error states as $e_1 = y_1 - x_1$ and $e_2 = y_2 - x_2$, the error state equations for each subsystem are as follows:

The first subsystem:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = 35.5 \sin(2t - 24) (e_1 - 0.001) - 0.5e_2 + 0.0355 \sin(2t) - \\ 0.024 + \Delta f(y_1, y_2) + U_1(t) \end{cases} \quad (9)$$

The second subsystem:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = 31.1542 \sin(2t - 30.9453) (e_1 - 0.5) - 0.5e_2 + \\ 17.0196 \sin(2t) - 13.1201 + \Delta f(y_1, y_2) + U_2(t) \end{cases} \quad (10)$$

The third subsystem:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = 31.1542 \sin(2t - 30.9453) (e_1 + 0.5) - 0.5e_2 - \\ 17.0196 \sin(2t) + 13.1201 + \Delta f(y_1, y_2) + U_3(t) \end{cases} \quad (11)$$

To obtain $U_j(t)$ where $j = 1, \dots, 3$, the fuzzy controller is proposed. The i th rule for the controller can be expressed as follows:

$$R^i : \text{if } e_1 \text{ is } k_1 \text{ and } e_2 \text{ is } k_2 \text{ then } u_{li} \equiv f_i(e_1, e_2) \quad (12)$$

where k_1 and k_2 are the input fuzzy sets, u_{li} is the output which is the analytical function $f_i(\cdot)$ of the input variables (e_1, e_2) .

Also $U_j(t)$ given as follows:

$$U_j = \frac{\sum_{i=1}^n \mu_i u_{li}}{\sum_{i=1}^n \mu_i}, \mu_i = \min(\mu_{k1}(e_1), \mu_{k2}(e_2)). \quad (13)$$

Using P, Z and N as input fuzzy sets representing 'positive', 'zero' and 'negative', respectively, we obtain the membership function.

According to error state equation a Lyapunov function is consider:

$$V = \frac{1}{2}(e_1^2 + e_2^2) \quad (14)$$

For stability, this condition must be met

$$\dot{V} = e_1 \dot{e}_1 + e_2 \dot{e}_2 < 0, \text{ so } \dot{e}_2 < -\frac{e_1 \dot{e}_1}{e_2}. \quad (15)$$

According the Lyapunov stability condition, the following case will satisfy all the stability conditions.

Case1. For $e_2 > 0$, the equation (14) becomes to $\dot{e}_2 < -e_1$.

It will be different for each subsystem. For the first subsystem we have:

$$35.5 \sin(2t - 24) (e_1 - 0.001) - 0.5e_2 + 0.0355 \sin(2t) - 0.024 + \Delta f(y_1, y_2) + u_{l1}(t) < -e_1$$

With consider $|\Delta f(y)| \leq \alpha$, the control signal is:

$$u_{l1}(t) = -e_1 - 35.5 \sin(2t - 24) (e_1 - 0.001) + 0.5e_2 - 0.0355 \sin(2t) + 0.024 - \alpha \quad (16)$$

It is including the three rules: $u_{l1}(t) = u_{l4}(t) = u_{l7}(t)$.

Case2. For $e_2 < 0$, the equation (14) becomes to $\dot{e}_2 > -e_1$.

So, like the previous case, the control signal as follows:

$$u_{l3}(t) = -e_1 - 35.5 \sin(2t - 24) (e_1 - 0.001) + 0.5e_2 - 0.0355 \sin(2t) + 0.024 + \alpha \quad (17)$$

It is including the three rules: $u_{l3}(t) = u_{l6}(t) = u_{l9}(t)$.

Case3. For $e_2 \in z, e_1 > 0$, because $e_1 + \dot{e}_2 = -\text{sgn}(e_2)$, the equation (14) becomes to $\dot{e}_2 < -\text{sgn}(e_2)$.

The control signal as follows:

$$u_{l2}(t) = -\text{sgn}(e_2) - 35.5 \sin(2t - 24) (e_1 - 0.001) + 0.5e_2 - 0.0355 \sin(2t) + 0.024 - \alpha \quad (18)$$

case4. For $e_2 \in z, e_1 < 0$, the equation (14) becomes to $\dot{e}_2 > -\text{sgn}(e_2)$.

The control signal as follows:

$$u_{l8}(t) = -\text{sgn}(e_2) - 35.5 \sin(2t - 24) (e_1 - 0.001) + 0.5e_2 - 0.0355 \sin(2t) + 0.024 + \alpha \quad (19)$$

case5. $e_2, e_1 \in z$, This condition is included in the other rules, and we define $u_{l5}(t) = 0$ in this rule.

In the same way we do for other subsystems. Finally, the overall control signal from equation (4) is obtained.

5. Simulation Result

In this section the proposed controller is applied to system. The initial values of the master system (6) and the slave system (7) are taken as and , respectively. is considered. In fig1, it shows that the slave system and the master system can reach synchronization with control operation. In addition, the time responses control input is shown in fig2, it shows that the resulting control input is continuous and smooth, and it is easy to implement in real physical system.

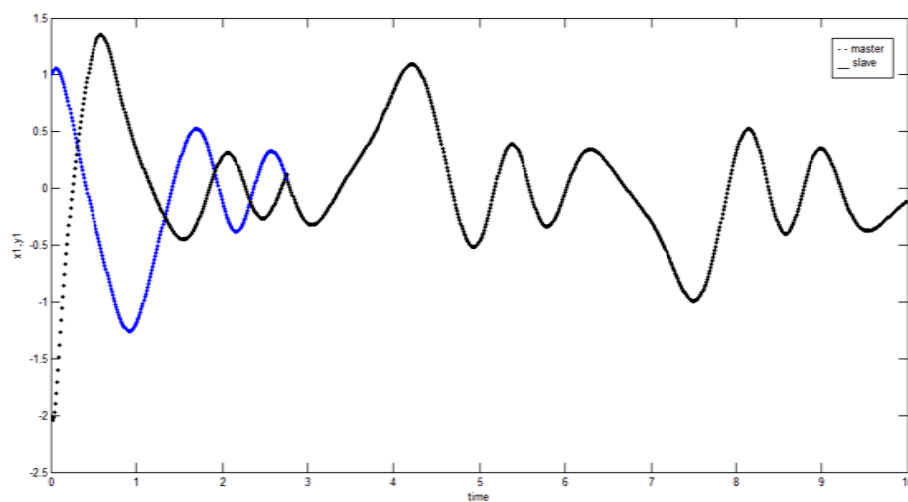


Fig. 1: Time responses of controlled chaotic gyro

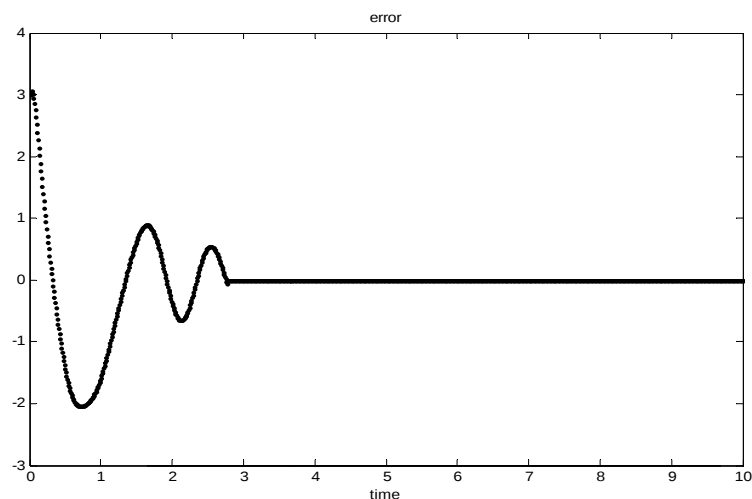


Fig. 2: Time responses control input synchronization system.

6. Conclusion

In this paper, TS fuzzy control scheme to synchronization of chaotic gyroscopes is proposed. For reducing the uncertainty in gyroscope dynamic model and time of the control actions a nonlinear model system is transformed into linear subsystems. Our numerical simulations have demonstrated the validity and feasibility of the proposed method.

References

- [1] H.K. Chen, (2002), Chaos and chaos synchronization of a symmetric gyro with linear-plus-cubic damping, *Journal of Sound and Vibration*, 255,pp. 719–740,.
- [2] R.V. Dooren, (2003),Comments on “chaos and chaos synchronization of a symmetric gyro with linear-plus-cubic damping”, *Journal of Sound and Vibration*, 268,pp.632-634.
- [3] Z.M. Ge, H.K. Chen(1996), Stability and chaotic motions of a symmetric heavy gyroscope, *Japanese Journal of Applied Physics* ,pp.1954–1965 .
- [4] Z.M. Ge, H.K. Chen, (1996), Bifurcations and chaos in a rate gyro with harmonic excitation, *Journal of Sound and Vibration*,pp. 107–117.
- [5] Z.M. Ge, H.K. Chen, H.H. Chen,(1996), The regular and chaotic motions of a symmetric heavy gyroscope with harmonic excitation, *Journal of Sound and Vibration*, pp.131–147.
- [6] X. Tong, N. Mrad, (2001), Chaotic motion of a symmetric gyro subjected to a harmonic base excitation, *Transactions of American Society of Mechanical Engineers, Journal of Applied Mechanics*, pp. 681–684.
- [7] S. A. Mohseni, S. M. Mohseni,(2008), Fuzzy Neural Networks Controller for a Chaotic Nonlinear Gyro Using Sliding-Mode Surfaces, *IEEE*,pp.1150-1153.
- [8] B. A. Idowu,(2008), Control and Synchronization of Chaos in Nonlinear Gyros via Back stepping Design, *International Journal of Nonlinear Science*, pp.11-19.
- [9] Leipnik RB, Newton TA,(1981), Double strange attractors in rigid body motion, *Phys Lett A*, pp.63–67.
- [10] Park CW, Lee CH, Park M, (2002), Design of an adaptive fuzzy model based controller for chaotic dynamics in lorenz systems with uncertainty. *Inf Sci*, pp.245–266.
- [11] M. A. Khanesar. M. Teshnehlab, (2012),Control and synchronization of chaotic systems using a novel indirect model reference fuzzy controller, *Springer-Verlag, Soft Comput* ,pp.914-926.

