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Type-2 Fuzzy Logic Control of Continuous Stirred Tank Reactor

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Abstract

This paper presents a Type-2 Fuzzy PI Controller applied to a CSTR (Continuous Stirred Tank Reactor). To describe the behavior of CSTR, mass and component balance equations have been developed and a non-linear CSTR plant has been modeled with the help of those equations. Through MATLAB/Simulink environment, an IT2 FLC has been designed and applied to the CSTR plant. The objective of the controller is to maintain the product concentration by adjusting one of the flow rates. The robustness analysis has also been performed on the IT2 FLC designed for the non linear CSTR plant by varying different parameters of the plant and by changing the footprint of uncertainty (FOU) of the IT2 FLC. The analysis has shown that IT2 FLC is generally more robust than their type-1 counter parts, as they are better able to cope with disturbances and uncertainties and eliminate oscillations. This is due to an additional degree of freedom provided by the footprint of uncertainty.

1. Introduction

In 1965, Professor L. A. Zadeh, from Berkeley University (USA), suggested an alternative for the conventional theory of sets, it was the fuzzy sets theory. It is much more flexible and similar to the theory of the possibilities. Basically a type-2 fuzzy set is a set in which we also have uncertainty about the membership function. Type-1 fuzzy sets are not able to directly model the uncertainties because their membership functions are totally crisp. On the other hand type-2 fuzzy sets (T2FS) are able to model the uncertainties because their membership functions are themselves fuzzy. Membership function of type-1 fuzzy sets (T1FS) are two dimensional, whereas membership function of type-2 fuzzy sets are three dimensional. It is the new third dimension of type-2 fuzzy sets that provides additional degrees of freedom that make it possible to directly model the uncertainties.

Type-2 fuzzy sets are difficult to understand and use because: (1) the three dimensional nature of type-2 fuzzy sets that makes them difficult to draw; (2) there is no simple collection of well-defined terms that let us effectively communicate about type-2 fuzzy sets and to then me mathematically precise about them; (3) derivations of the formulas for the union, intersection and complement completely rely on using Zadeh's Extension principle which itself is a difficult concept; (4) using type-2 fuzzy sets is computationally more complicated than using type-1 fuzzy sets. A type-1 fuzzy set is characterized by a two-dimensional, membership function whereas a type-2 fuzzy set is characterized by a three-dimensional membership function. The new third dimension provides additional degree of freedom that makes it possible to directly model and handle uncertainties.

This work presents the design and implementation of IT2-FLC for a non linear plant of CSTR. The CSTR plant which has highly nonlinear characteristics is first modelled with the help of its mass and component balance equations in MATLAB/Simulink. Furthermore the paper focuses on the robustness analysis of IT2 FLSs.

The remaining of this paper is organized as follows: Chapter-3 presents the some background theories on Type-2 Fuzzy logic and also explains the general structure of an Interval Type-2 Fuzzy Logic System. The non-linear plant chosen for the analysis i.e. Continuous Stirred Tank Reactor (CSTR) is described in Chapter-3. Chapter-4 demonstrated the design and implementation of IT2-FLC applied to CSTR. The performance of IT2-FLC applied to CSTR and its robustness check is shown in Chapter-5. Chapter-6 gives some conclusions.

2. Type-2 Fuzzy Logic Control

The Fuzzy Logic Systems consists of rules. Very often, the knowledge that is used for building these rules is uncertain. Such uncertainty and in-exactness leads to rules in which the antecedents and/or consequents are uncertain, which transforms into uncertain antecedent or consequent membership functions. In Type-1 fuzzy systems the membership functions are type-1 fuzzy sets, and they are unable to directly handle such uncertainties. In this chapter, we describe, type-2 fuzzy systems, in which the antecedent or consequent membership functions are type-2 fuzzy sets. Such sets are fuzzy sets the membership functions are also fuzzy; they are very useful in situations where it is difficult to determine an exact membership function for a fuzzy set.

An interval type-2 fuzzy set (IT2FS) $\tilde{A}$ is characterized as :

$$\tilde{A} = \int_{x\in X} \int_{u\in Jx\subseteq[0,1]} 1/(x,u) = \int_{x\in X} [\int_{u\in Jx\subseteq[0,1]} 1/u]/x \tag{1}$$

where x, the primary variable, has domain $X; u \in U$, the secondary variable, has domain J_x at each $x \in X$; J_x is called the primary membership of x; and, the secondary grades of $\tilde{A}$ all equal 1. Note that Eq. (1) means
$\tilde{A} : X \rightarrow \{[a,b]: 0 \leq a \leq b \leq 1\}$. Uncertainty about $\tilde{A}$ is conveyed by the union of all the primary memberships, which is called the footprint of uncertainty (*FOU*) of $\tilde{A}$ Fig.1, i.e.

$$FOU(\tilde{A}) = \bigcup_{\forall x \in X} J_x = \{(x,u) : u \in J_x \subseteq [0,1]\} \qquad (2)$$

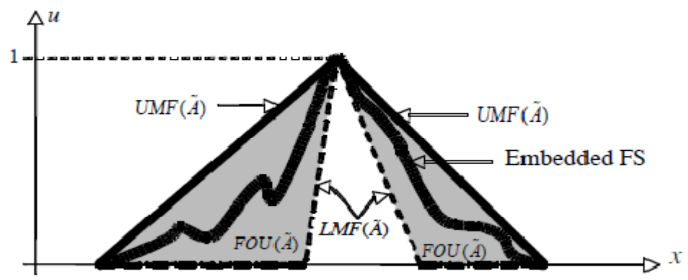

Figure 1: FOU (Shaded), LMF (dashed), UMF (solid) and an embedded FS (Wavy line) for IT2FS Ã.

The upper membership function (UMF) and lower membership function (LMF) of $\tilde{A}$ are two type-1 MFs that bound the FOU Fig. 1. The UMF is associated with the upper bound of FOU ($\tilde{A}$) and is denoted by $\overline{\mu}_{\tilde{A}}(x), \forall x \in X$ and the LMF is associated with the lower bound of FOU (Ã) and is denoted by $\underline{\mu}_{\tilde{A}}(x), \forall x \in X$ i.e.

$$\overline{\mu}_{\tilde{A}}(x) \equiv \overline{FOU(\tilde{A})}, \forall x \in X \qquad (3)$$

Note that J_x is an interval set, i.e.

$$J_x = \{(x,u) : u \in [\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)] \qquad (4)$$

So that FOU(Ã) in Eq. (2) can also be expressed as

$$FOU(\tilde{A}) = \bigcup_{\forall x \in X} [\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)] \qquad (5)$$

A T2 FLS is depicted in Fig. 2. It is very similar to the T1 FLS, the major structural difference being that the defuzzifier block of a T1 FLS is replaced by the *output processing* block in a T2 FLS. That block consists of type-reduction followed by defuzzification. Type-reduction maps a T2 FS into a T1 FS, and then defuzzification, as usual, maps that T1 FS into a crisp number. Here we assume that all the antecedent and consequent fuzzy sets in rules are T2; however, this need not necessarily be the case in practice. All results remain valid as long as just one FS is T2. This means that a FLS is T2 as long as any one of its antecedent or consequent (or input) FSs is T2.

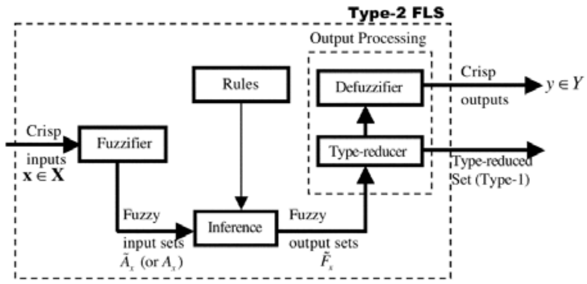

Figure 2: Type-2 Fuzzy Logic System.

3. Continuous Stirred Tank Reactor

Continuous Stirred Tank Reactor (CSTR) is highly non-linear process exhibiting stable and unstable steady state at its different regions of operation. Control of CSTR in the complete range is a mind boggling problem. In a continuous-flow stirred-tank reactor (CSTR), reactants and products are continuously added and withdrawn. In practice, generally a mechanical or hydraulic agitation is required to achieve a uniform composition and temperature. Because the compositions of mixtures leaving the CSTR are those within the reactor, the reaction driving forces, the reactant concentrations, are necessarily low. However, the low driving forces makes better control over the rapid endothermic and exothermic reactions. Whenever high conversions of reactants are needed, several CSTRs can be used in series. Equally better results can be obtained by dividing a single vessel into smaller compartments while minimizing back-mixing or short-circuiting.

To ensure the successful operation of a continuous stirred tank reactor (CSTR) it is necessary to understand their dynamic characteristics. A good understanding will ultimately enable us to achieve an effective control systems design. To describe the dynamic behavior of a CSTR mass, component and energy balance equations must be developed. This requires a good understanding of the functional expressions that can completely describe chemical reaction. A chemical reaction will create new components while simultaneously reducing the reactant concentrations. The reaction may produce heat or may require energy to proceed. The schematic diagram of the CSTR is shown in Fig 3. One the inlet stream consists of pure component A with molar concentration C_{b1} and flow rate w_1, and the other inlet stream consists of component B with molar concentration C_{b2} and flow rate w_2.

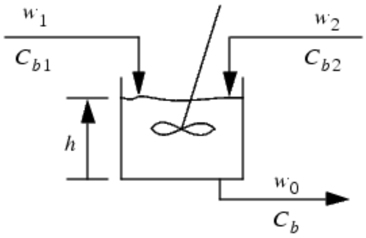

Figure 3: Process diagram of CSTR.

$$\frac{dh}{dt} = w_1(t) + w_2(t) - 0.2\sqrt{h} \tag{6}$$

$$\frac{d(C_b(t))}{dt} = \left(C_{b1} - C_b(t)\right)\frac{w_1(t)}{h(t)} + \left(C_{b2} - C_b(t)\right)\frac{w_2(t)}{h(t)} - \frac{k_1 C_b(t)}{(1+k_2 C_b(t))^2} \tag{7}$$

Where Eqs (7-8) represents mass and component balance equation of CSTR respectively.

4. Interval Type-2 Fuzzy Logic Control of CSTR

An Interval Type-2 Fuzzy Logic Controller (IT2-FLC) for a non-linear CSTR plant. It is well known that processes in a CSTR (and many other chemical reactors and processes) present considerable amount of nonlinearity with respect to its different operating regions. To design a fuzzy controller that gives satisfactory performance for different regions of nonlinearities, an IT2-FLC with two inputs is used, to allow different control strategies to be designed in different regions. The inputs are control error (e) and change in the control error (Δe) with an output control action (u).

u = IT2-FLC (e, Δe)

Where, IT2-FLC(.) stands for the nonlinear relationship of an interval type-2 fuzzy controller.

The rule base of an IT2-FLC designed for CSTR is summarized in Table-1. The linguistic variables used in the membership functions are: NB-Negative Big, NM-Negative Medium, NS-Negative Small, ZE-Zero Error, PS-Positive Small, PM-Positive Medium, PB-Positive Big.

Table 1: Rules for Interval Type-2 Fuzzy Logic Controller

u(t) Controller Output	Error [e(t)]							
		NB	**NM**	**NS**	**ZE**	**PS**	**PM**	**PB**
Error Change [Δe(t)]	**NB**	NB	NB	NB	NB	NM	NS	ZE
	NM	NB	NM	NM	NM	NS	ZE	PS
	NS	NB	NM	NS	NS	ZE	PS	PM
	ZE	NB	NM	NS	ZE	PS	PM	PB
	PS	NM	NS	ZE	PS	PS	PM	PB
	PM	NS	ZE	PS	PM	PM	PM	PB

	PB	ZE	PS	PM	PB	PB	PB	PB

The fuzzy rule base can be read as follows:
Rule-1: If e is NB and Δe is NB, then u is NB
Rule-2: If e is NM and Δe is NB, then u is NB, and so on….

For implementing an Interval Type-2 Fuzzy Controller it is necessary to define the fuzzy membership functions associated with the controller inputs: the control error (e) and the change in the control error (Δe) and with the control action (u) based on prior knowledge about the process.

For the rules mentioned in Table-1, Fig. 4 shows the membership functions selected for this problem.

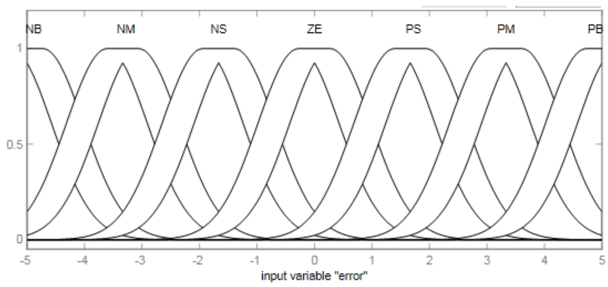

Figure 4. Membership functions for the input error ‘e’ and change in error ‘Δe’ and output control action ‘u’

Using MATLAB/Simulink, it is possible to implement a CSTR and further designing an IT2-FLC for CSTR. For the completion of the proposed process simulation in MATLAB/Simulink, first the CSTR have to be modelled with the help of its mass balance and component balance equations that we have derived earlier. Fig. 5 shows a Simulink model of a CSTR, where w_1 is the flow rate of concentrated feed C_{b1} and w_2 is the flow rate of diluted feed C_{b2}. The input concentration are set to C_{b1}=24.9 and C_{b2}=0.1. The constants associated with the rate of consumption are $k_1 = 1$ and $k_2 = 1$. To simplify the demonstration, set $w_2(t) = 0.1$.

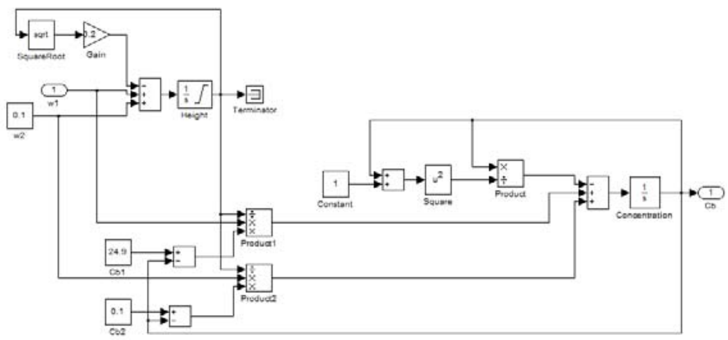

Figure 5. Matlab/Simulink representation of a CSTR model consisting of mass balance and component balance equation

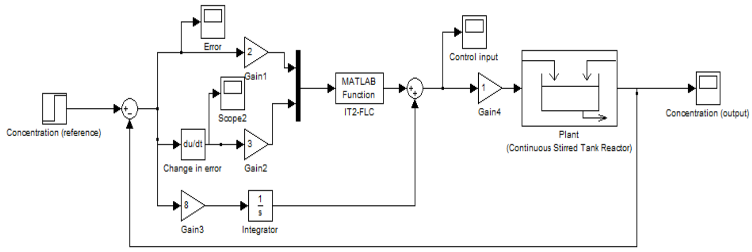

Figure 6. Simulink representation of a CSTR with IT2 FLC.

Fig.6 Shows the Simulink representation of CSTR with an IT2-FLC. The objective of the controller is to maintain the product concentration by adjusting the flow $w_1(t)$. The fuzzy controller is implemented with error 'e' and change in error 'Δe' as its input, and control action 'u' as its output. Gain1 and gain2 are scaling factors for the input and gain4 act as scaling factor for the output. The input and output scaling factors are determined by trial and error method. These scaling factors are tuning parameters for the fuzzy controller. The integral term is then added to the output of fuzzy inference system to improve steady state response and to reduce the steady state error.

5. Results and Discussions

The IT2-FLC controller designed in this work shows good response in the entire operating region of the CSTR which has highly non-linear characteristics. The controller operates in a regulated manner so that any disturbance to the system is eliminated. The controller was designed to control the product concentration of CSTR by varying the flow rate of one of the inlet stream. The IT2-FLC designed for CSTR as shown in Fig. 7 showed best performance for both set point tracking and regulatory conditions.

The performance of the IT2-FLC implemented in MATLAB Simulink, when applied to a CSTR plant is shown in the Fig.7 with set point as 23 which is the desired concentration. The result shows that the response of the CSTR i.e. concentration settles down at 49.22 seconds, with an percentage overshoot of 1.5% and has rise time of 3.84 seconds.

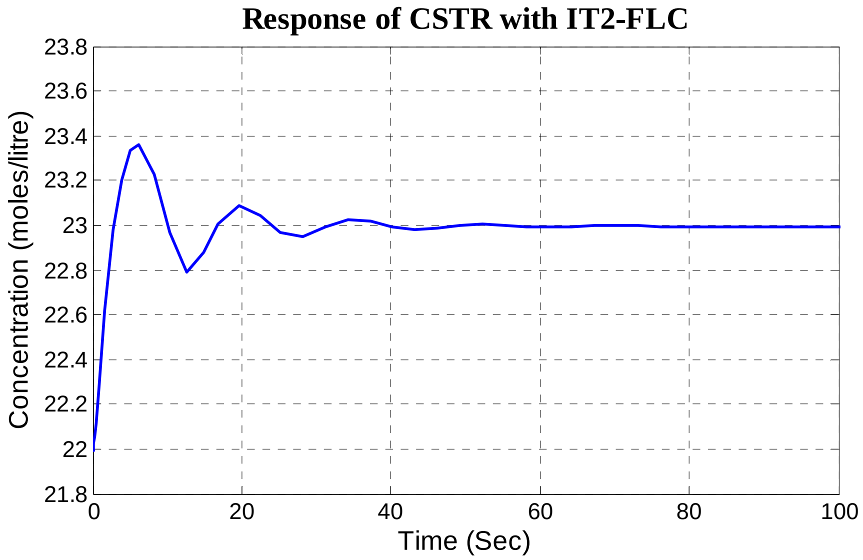

Figure 7. Response of CSTR with IT2 FLC

The performance of the IT2-FLC is tested by varying the reference value of concentration in between 21 and 23.

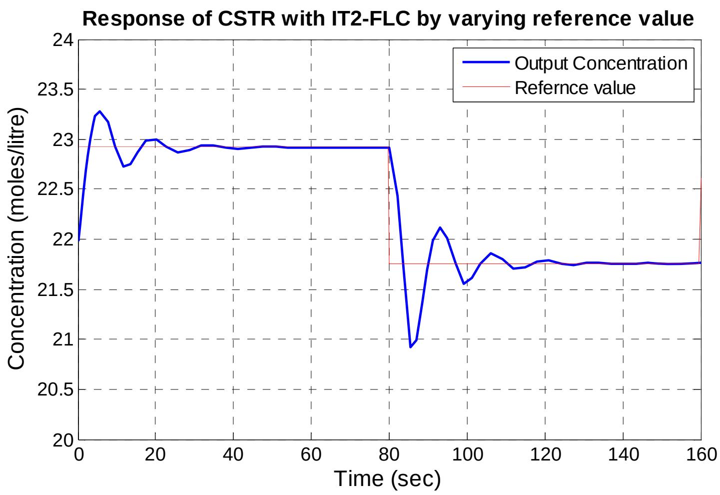

Figure 8. Response of CSTR with IT2-FLC by varying reference value.

In order to examine, the robustness of IT2-FLC applied to CSTR, the performance of IT2-FLC is analyzed by varying the FOU. Fig.9 shows the response of IT2-FLC at two different FOUs i.e. at 0.3 and 0.5. The Fig.9 has clearly demonstrated that how a change in FOU size would influence the performance of an IT2 fuzzy PI controller. It is observed that, as we increase the FOU, the ability of an IT2-FLC to eliminate oscillations about the set point increases. A larger FOU increases the damping of the IT2-FLC controller, and hence reduces overshoots and oscillations.

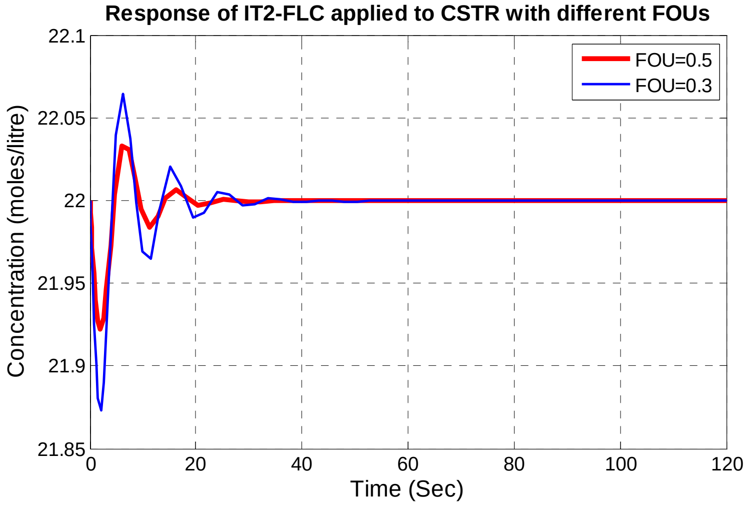

Figure 9: Response of IT2-FLC with different FOUs.

Further, the robustness of IT2-FLC is demonstrated by varying the parameters of the CSTR plant. Fig.10 shows the performance of IT2-FLC at different values of parameters of the plant. The parameters that are varied for this analysis are α, C_{b1},C_{b2}, w_2 .

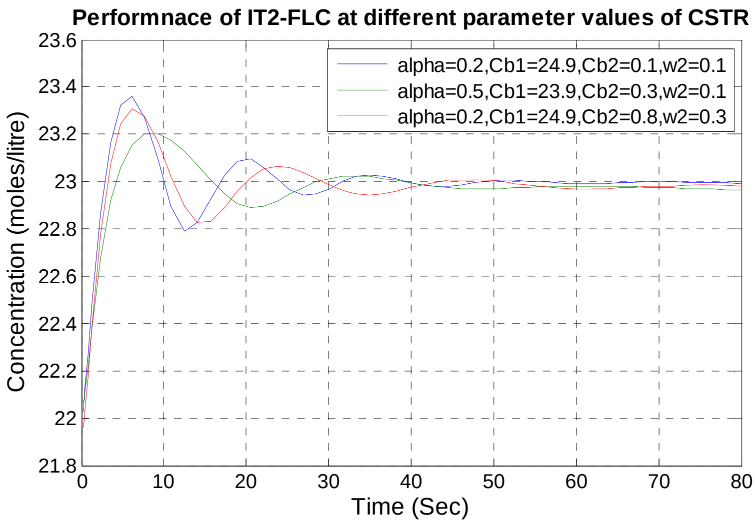

Figure 10. Performance of IT2-FLC at different parameter values of CSTR.

6. Conclusions

The CSTR plant which has highly nonlinear characteristics is first modelled with the help of its mass and component balance equations in MATLAB/Simulink. The controller was designed to control the product concentration of a CSTR. Simulation results show good dynamic performance of IT2-FLC over the entire operating region of CSTR. The IT2-FLC also shows best performance for both set point tracking and regulatory conditions even when the reference value of concentration is changed. The results obtained show that the performance of this IT2-FLC is very adequate to control nonlinearities in the processes.

This work also focuses on the robustness analysis of IT2 FLSs and it was concluded that IT2-FLC are more robust than their Type-1 counter part because of the presence additional degree of freedom provided by FOU. The response of the IT2-FLC shown in Fig. 9 and 10 reveals that IT2-FLC is better able to cope with uncertainty and eliminate steady-state oscillations, as it performs satisfactorily even if the parameters of CSTR is varied and FOU is changed. But a larger FOU increases the damping of the controller, and hence reduces overshoots and oscillations.

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