Advance in Electronic and Electric Engineering.
ISSN 2231-1297, Volume 3, Number 2 (2013), pp. 137-144

http://www.ripublication.com/aeee.htm

Designing Robust Control by Sliding Mode Control Technique

Chintu Gurbani[1], Dr. Vijay Kumar[2]

[1]E&CE Dept., Indian Institute of Technology, Roorkee
G-56, Govind Bhawan, IIT Roorkee, Roorkee, Uttarakhand, India.
[2]E&CE Dept., Indian Institute of Technology, Roorkee.

Abstract

The control of dynamic systems in presence of parametric matched uncertainties and disturbances is a common problem to deal with when considering real plants. The effect of these uncertainties on the system dynamics should be carefully taken into account since they can reduce the performance or even cause system instability. This paper deals with the designing of a controller using different types of sliding mode control strategies. Sliding mode control utilizes discontinuous control laws to drive the system state trajectory onto a specified surface in the state space, the so called sliding or switching surface, and to keep the system state on this manifold for all the subsequent times. In order to achieve the control objective, the control input must be designed with an authority sufficient to overcome the uncertainties and the disturbances acting on the system. Sliding mode control is a technique to make any linear or nonlinear system completely insensitive to parametric uncertainty and external disturbances. In this paper, an analysis among two sliding mode control strategies, conventional control and quasi control, is being done. The biggest problem which we face in the path of implementing sliding mode control is control chattering. Several methods to suppress chattering can be used. The performance of the designed controllers is then studied by means of simulation using MATLAB and SIMULINK.

1. Introduction

One of the common problems which we face in control of dynamical systems is the presence of uncertainties and disturbances when considering real plants. These disturbances sometime degrade the performance or even cause instability. Due to these reasons, during recent years, the problem of controlling dynamical systems in presence of uncertainty conditions has become an important subject of research. As a result, considerable progresses have been attained in robust control techniques, such as nonlinear adaptive control, model predictive control, backstepping, sliding model control and others. These techniques are capable of guaranteeing the attainment of the control objectives in spite of modelling errors or parameter uncertainties affecting the controlled plant. Among the existing methodologies, the Sliding Mode Control (SMC) technique turns out to be characterized by high simplicity and robustness. Essentially, SMC utilizes discontinuous control laws to drive the system state trajectory onto a specified surface in the state space, the so called sliding or switching surface, and to keep the system state on this manifold for all the subsequent times. In order to achieve the control objective, the control input must be designed with an authority sufficient to overcome the uncertainties and the disturbances acting on the system. The main advantages of this approach are two. First, while the system is on the sliding manifold it behaves as a reduced order system with respect to the original plant and second, the dynamic of the system while in sliding mode is insensitive to model uncertainties and disturbances. This latter property of invariance towards so called matched uncertainties is the most distinguish feature of sliding mode control and makes this methodology particular suitable to deal with uncertain linear and nonlinear systems. Designing of controller using sliding mode control can be done by a conventional method insuring stability of the system dynamics which is explained in next section.

2. Designing Sliding Mode Control

2.1 Conventional sliding mode control (CSMC)

Let the control law u be the control that drives the state variables x_1, x_2 to the sliding surface in finite time t_r, and keeps them onto the surface thereafter in the occurrence of the bounded disturbance $f(x_{1,}x_2, t)$.

Let the system be,

$$\dot{x_1} = x_2 \; x_1(0) = x_{10}$$
$$\dot{x}_2 = u + f(x_1, x_2, t) \; x_2(0) = x_{20} \quad (1)$$

Here the disturbance $f(x_1, x_2, t)$ may contain dry and viscous friction as well as any other unidentified resistance forces, and is believed to be bounded, i.e. $f(x_1, x_2, t) \leq \mathrm{L} > 0$.

Our desired compensated dynamics will be one having no effect of the disturbance $f(x_1, x_2, t)$.

Let us introduce a new variable known as sliding variable in the state space of the system,

$$\sigma = \sigma\left(x_{1,}x_1\right) = x_2 + cx_1, c > 0 \quad (2)$$

In order to attain asymptotic convergence of the state variables x_1, x_2 to zero, i.e. $\lim_{t\to\infty} x_1, x_2 = 0$ in the presence of the bounded disturbance $f(x_1, x_2, t)$,we have to take the sliding variable to zero in finite time by the means of control u. This job can be achieved by applying Lyapunov function techniques to the σ dynamics using equations (1) and (2).

$$\dot{\sigma} = cx_2 + f(x_1, x_2, t) + u, \sigma(0) = \sigma_0 \tag{3}$$

Equation (2) and (3) rewritten in the form $\sigma = x_2 + cx_1 = 0, c > 0$ correspond to a straight line in the state space of Eq. (1) and are referred to as a Sliding Surface.

For the σ dynamics (3) we introduce a Lyapunov function of the form

$$V = \frac{1}{2}\sigma^2 \tag{4}$$

In order to give asymptotic stability to Eq. (3) about the equilibrium point σ=0,

a) $\dot{V} < 0 \; for \; \sigma \neq 0$
b) $\lim_{|\sigma| \to \infty} V = \infty$

Condition (b) is satisfied by V in Eq. (4). In order to achieve finite-time convergence, condition (a) can be modified to be

$$\dot{V} \leq -\alpha V^{1/2}, \alpha > 0 \tag{5}$$

Integrating Eq. (5) in the interval **0** to **t,** we obtain,

$$V^{1/2}(t) \leq -\frac{1}{2}\alpha t + V^{1/2}(0) \tag{6}$$

Consequently, V (t) reaches 0 in a finite time t_r that is bounded by

$$t_r \leq \frac{2V^{1/2}(0)}{\alpha}. \tag{7}$$

So, a control u that is derived to satisfy Eq. (5) will drive the variable σ to zero in finite time and will keep it at zero afterwards. The derivative of V is computed as

$$\dot{V} = \sigma\dot{\sigma} = \sigma(cx_2 + \; f(x_1, x_2, t) + u) \tag{8}$$

Assume $u \; = -cx_2 \; + \; v$ and substituting it into Eq. (8), we obtain,

$$\dot{V} = \sigma(f(x_1, x_2, t) + \; v) = \sigma f(x_1, x_2, t) + \sigma v \leq |\sigma|L + \sigma v$$

Selecting $v = -\rho sign(\sigma)$ where

$$\text{Sign(x)} = \begin{cases} 1 \text{ if } x > 0 \\ -1 \text{ if } x > 0 \end{cases}$$
$$\text{And sign (0)} \in [-1, 1] \tag{9}$$

With $\rho > 0$ and substituting in expression of $\dot{V}$, we get

$$\dot{V} \leq |\sigma|L - |\sigma|\rho = -|\sigma|(\rho - L) \tag{10}$$

Taking into account Eq. (4) and (5) we get,

$$\dot{V} \leq -\alpha V^{1/2} = -\frac{\alpha}{\sqrt{2}}|\sigma|, \alpha > 0 \tag{11}$$

Finally, the control gain $\rho = L + \frac{\alpha}{\sqrt{2}}$
Thus, a control law u that drives σ to zero in finite time is

$$\mathrm{u} = -\mathrm{cx}_2 - \rho\mathrm{sign}(\sigma) \tag{12}$$

The control gain of Eq. (12) is designed to compensate for the bounded disturbance $f(x_1, x_2, t)$ while the second term $\frac{\alpha}{\sqrt{2}}$ is accountable for determining the sliding surface reaching time given by Eq. (7). The larger the value of α, the shorter reaching time. Condition +(5) is equivalent to

$$\sigma\dot{\sigma} \leq -\frac{\alpha}{\sqrt{2}}|\sigma| \tag{13}$$

Equation (13) is often termed the reachablity condition and is essential condition for sliding mode to occur. Meeting the existence condition (13) means that the state trajectory of the system in Eq. (1) is focused towards the sliding surface and remains on it afterwards.

The phase plane trajectory, so obtained from above sliding mode control technique suffers from a zigzag motion which has a small amplitude and high frequency when sliding mode occurs. This effect is known as chattering. In many real time practical control systems it is required to avoid the chattering effect.

2.2 Chattering phenomenon

In the design of sliding-mode controllers for realistic applications, it is essential to determine a proper sliding surface so that the output deviations can be reduced to a acceptable level. The sliding-mode control, despite the advantages of simplicity and robustness, generally suffer from the problem, called chattering, which is a very high-frequency oscillation of the sliding variable around the sliding manifold. The chattering is undesirable for the real systems and actuator since it may direct to actuator breakdown and unreasonably large control signal. In practice, the occurrence of time delays in actuators and in many industrial processes, such as transportation lag, time lag, time delay, physical limitations and dead time cannot switch at an infinite frequency along the sliding surface as demand by the assumption of sliding-mode control algorithms. Many approaches have been projected to overcome chattering phenomenon. It may be prevail over by smoothing out the control discontinuity. Frequently preferred approaches are to substitute the control by a saturation function, hyperbolic functions, sigmoid functions, and hysteresis saturation functions [4].

2.3 Quasi-sliding mode control (QSMS)

One solution to eliminate chattering is to approximate the discontinuous function $v(\sigma) = -\rho \text{sign}(\sigma)$ by some continuous and smooth function. For example, it could be replaced by a "sigmoid function".

$$\text{sign}(\sigma) \approx \frac{\sigma}{|\sigma|+\varepsilon} \tag{14}$$

Where ε is a small positive scalar value. It can be observed that for $\sigma \neq 0$ point-wise

$$\lim_{\varepsilon \to 0} \frac{\sigma}{|\sigma|+\varepsilon} = sign(\sigma) \tag{15}$$

$$u = -cx_2 - \rho \frac{\sigma}{|\sigma|+\varepsilon} \tag{16}$$

Due to smooth control law the sliding variable and the state variables do not converge to zero, but in its place converge to domains in a vicinity of the origin due to the consequence of the disturbance. This smooth control law is known as quasi-sliding mode control [3].

2.4 Simulation

In this section, the performance of the projected methods are shown by applying it to following system

$$\dot{x}_1 = x_2 \; x_1(0) = 1$$
$$\dot{x}_2 = u + f(x_1, x_2, t) \; x_2(0) = -2$$

The control gain ρ=2, parameter c=1.5, and the bounded disturbance is $f(x_1, x_2, t) = \sin(618t)$.

Using conventional sliding mode control, the results of simulation are presented in figures 1-3

Using quasi-sliding mode control, the results of simulation are presented in figures 4-6

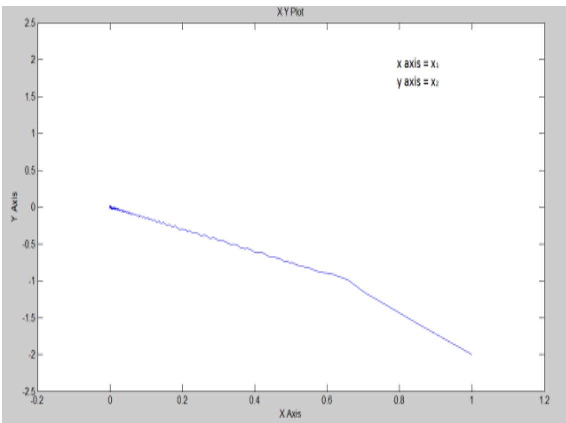

Figure 1: Phase plane trajectory using CSMC

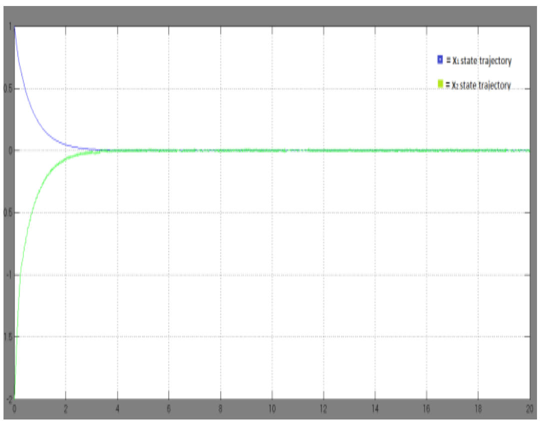
Figure 2 convergences of state trajectories using CSMC.

With Initial Condition, $x_1(0) = 1\, x_2(0) = -2$ Simulation is done using MATLAB and Simulink to verify the controller. The simulation results are shown in Fig. 1, Fig. 2 and Fig. 3. From the phase plane trajectory plot, we see that the trajectory starts from the initial points (1, -2), move towards the switching surface $x_1 and\ x_2$, then slide along the surface to reach the equilibrium point x = 0. According to Fig. 2, we can see that both signal x_1 and x_2 reach 0 after about 4 seconds. Also, noted from the x_2 plot, we could see that the trajectory reaches the switching surface when the time is approximately t_r = 3.3 seconds. However, for the control signal of the system, this control law has the drawback that the control signal chatters when the system trajectory is moving on the switching surface (refer Fig: 1)

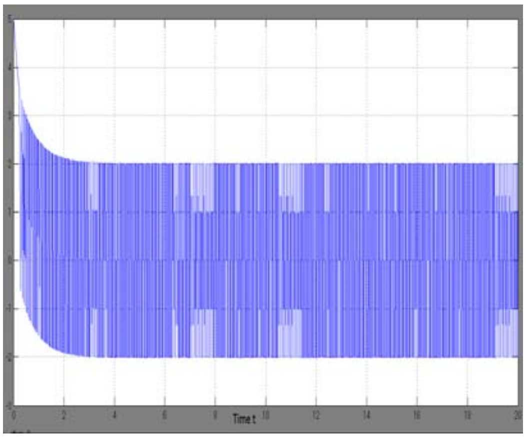

Figure 3 conventional sliding mode control law u

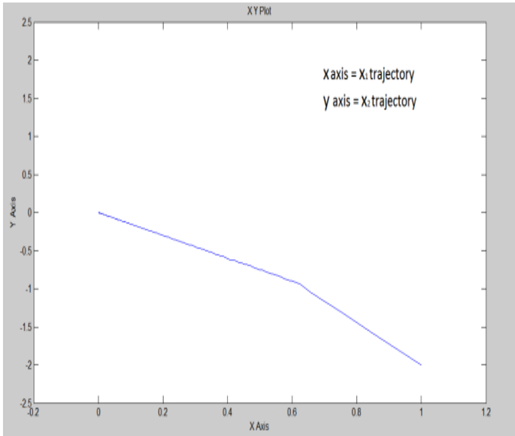

Figure 4: phase plane trajectory using quasi-SMC

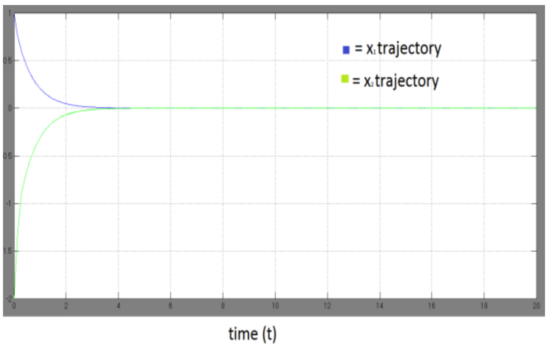

Figure 5: Convergences of trajectories using QSMC

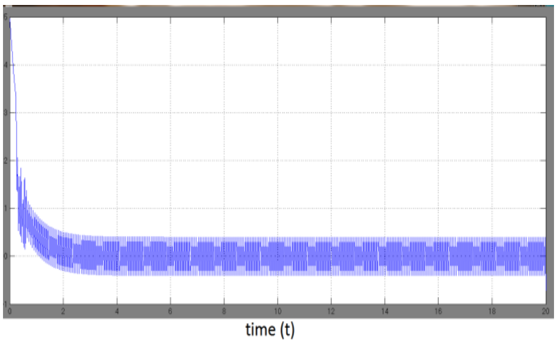

Figure 6: Quasi sliding mode controls

With Initial Condition, $x_1(0) = 1\, x_2(0) = -2$ Simulation is done using MATLAB and Simulink to verify the quasi sliding mode controller. ε=0.01. The simulation results are shown in Fig. 4, Fig. 5 and Fig. 6. From the phase plane trajectory plot, we see that the trajectory starts from the initial points (1, -2), move towards the switching surface $x_1 and\ x_2$, then slide along the surface to reach the equilibrium point x = 0. According to Fig, 5 we can see that both signal x_1 and x_2 reach 0 after about 4 seconds. Chattering is considerably eliminated as seen in phase plane trajectory figure 4 and a smooth trajectory is obtained.

3. Conclusion

This paper highlights the basics of sliding mode control, its control strategies, how it is applied, and its outcomes. It deals with the very basics of sliding motion, the presence of sliding surface and its control. Here most of the matter focuses on guaranteeing the robustness of sliding mode in the presence of practical engineering constraints and realities. Further conventional sliding mode technique can be used to design a robust controller for any linear system which will provide an attractive feature of being completely insensitive to parametric uncertainty and external disturbances during sliding mode. Further quasi-sliding mode control is explained which suppress the chattering occurring in conventional sliding mode control. An example is illustrated and the performance of both methods has been illustrated.

References

[1] Roy Smith and Andey Packard (1997), Article is from book, "Notes on Robust Control" spring, Chapter 1, pp 1-9.

[2] Vadim Utkin (1992), "sliding mode control" IEEE Transactions on Control Systems.

[3] Yuri Shtessel, Cristopher Edwards, Lionid Fridmen (1992), Article is from book "Sliding Mode Control and Observation" 3rd Ed. Chapter 1, pp 9-20.