

## Sensitivity Statistical Analysis of a Fractional Nonlinear Model of a Novel Coronavirus (COVID-19)

Salem Mubarak Al-Zahrani<sup>\*1</sup>, Fath E. I. Elsmih<sup>2</sup>, Kaled Salem Al-Zahrani<sup>3</sup>,  
Sayed Saber<sup>4</sup>

<sup>1</sup>*Faculty of Arts and Science in Almandaq, Al-Baha University, Saudi Arabia.*

<sup>2</sup>*Department of Mathematics, Faculty of Sciences and Arts in Baljurashi, Al-Baha University, Saudi Arabia.*

<sup>2</sup>*Department of Mathematics, Faculty of Sciences, Peace University, Sudan.*

<sup>3</sup>*Faculty of Arts and Science in Almandaq, Al-Baha University, Saudi Arabia.*

<sup>4</sup>*Department of Mathematics and Computer Science, Faculty of Science, Beni-Suef University, Egypt.*

<sup>4</sup>*Faculty of Sciences and Arts in Baljurashi, Al-Baha University, Saudi Arabia.*

### Abstract

In this paper, we state a simplified nonlinear fractional-order mathematical model to describe the dynamics of the novel coronavirus (COVID-19). The design of the fractional-order mathematical model is described in terms of four categories susceptible (S), infected (I), treatment (T), and recovered (R), namely SITR model with fractals parameters. The aim of our study is to compute the basic reproduction number  $\mathcal{R}_0$ . Sensitivity statistical analysis was carried out to see how changes in parameters affect the initial transmission of the Covid-19 disease. Finally, different techniques are introduced, and the model evolution is drawn up in different cases based on the rate of contact between individuals, which can be useful in reducing infection.

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<sup>\*</sup>Corresponding Author.

## 1. INTRODUCTION AND MODEL DESCRIPTION

In December 2019, a new virus belonging to the coronavirus strain has been discovered in Wuhan, China, this virus has attracted world-wide attention and it spread rapidly in the world, reaching nearly 216 countries in the world in November 2020.

Fractional differential equations are equations where the order of the derivative is a real or complex number. Its first appearance is in a letter written to Guillaume de l'Hôpital by Gottfried Wilhelm Leibniz in 1695. In recent years researchers have been interested in studying some real problems in various fields using fractional calculus, such as epidemiological models (see [1]-[10]), and others.

In this paper, we study the fractional incommensurate Sitr (susceptible, infections, treatment and removed) COVID-19 model with nonlinear saturated incidence rate using Caputo fractional derivatives. The novel COVID-19 interaction that we modeled in a simplified way is based on four mechanisms: susceptible ( $S$ ), infected ( $I$ ), treatment ( $T$ ) and recovered ( $R$ ). In addition, the susceptible ( $S$ ) is divided in two categories  $S_1(t)$  and  $S_2(t)$ . The state variables  $S_1(t)$ ,  $S_2(t)$ ,  $I(t)$ ,  $T(t)$  and  $R(t)$  represent, respectively, the densities of people who are yet not infected, the densities of not infected old age or serious diseased people, the infected people which are infected with this serious disease at the time  $t$ , the treatment of this virus, and the recovery of those people who recovered from this serious disease at time  $t$  by using these precautionary measures and it is noticed that a large number of such category exist. In this work, a fractional order (COVID-19) Sitr model with nonlinear incidence rate is considered in the sense of Caputo derivative  $D^\nu$ ,  $0 < \nu \leq 1$ , and is given by

$$\begin{aligned} D^\nu S_1(t) &= B - \beta I(t)S_1(t) - \delta \beta T(t) - S_1(t), \\ D^\nu S_2(t) &= B - \beta I(t)S_2(t) - \delta \beta T(t) - S_2(t), \\ D^\nu I(t) &= -\mu I(t) + \beta I(t)(S_1(t) + S_2(t)) + \delta \beta T(t) - \alpha I(t) + \sigma I(t), \\ D^\nu T(t) &= \mu I(t) - \alpha T(t) - \rho T(t) + \varepsilon T(t) + \psi T(t), \\ D^\nu R(t) &= -\alpha R(t) + \rho T(t), \end{aligned} \quad (1.1)$$

with initial data  $S_1(0) = I_1$ ,  $S_2(0) = I_2$ ,  $I(0) = I_3$ ,  $T(0) = I_4$ , and  $R(0) = I_5$ . The biological meanings of the parameters in the model (1.1) are existed in Table 1.

Forward sensitivity analysis has long been an important part of mathematical modeling, but it gets more difficult to apply as a model becomes more complicated. The sensitivity analysis of a parameter known as the basic reproduction number, or  $\mathcal{R}_0$ , has dominated the focus of ecological modelers in particular for infectious disease models. The biological definition of  $\mathcal{R}_0$ , on the other hand, is well established. (see [1]-[10])

Parameters	Description
$B$	Natural birth rate
$\beta$	Contact rate
$\delta$	Reduce infection by treatment
$\alpha$	Death rate
$\mu$	Recovery rate
$\sigma$	Dry cough, fever and tiredness rate
$\rho$	Infection rate for treatment
$\varepsilon$	Sleep rate
$\psi$	Healthy food rate

Table 1: Parameters description for the novel (COVID-19) Sitr model

## 2. THE BASIC REPRODUCTION NUMBER

For  $I = 0$ , we can easily obtain the infection-free equilibrium  $P_0 = (\frac{B}{\alpha}, \frac{B}{\alpha}, 0, 0, 0)$ . To obtain and stabilize the endemic-equilibrium point, one assumes that

$$\begin{cases} D^\nu S_1(t) = 0, \\ D^\nu S_2(t) = 0, \\ D^\nu I(t) = 0, \\ D^\nu T(t) = 0, \\ D^\nu R(t) = 0. \end{cases}$$

For  $I^* > 0$ , the unique endemic-equilibrium point is:  $P^* = (S_1^*, S_2^*, I^*, T^*, R^*)$ , where

$$\begin{aligned} S_1^* &= \frac{1}{2\beta\mathcal{R}_0(\alpha + \rho - \varepsilon - \psi)} + \frac{B}{\alpha}, \\ S_2^* &= S_1^*, \\ I^* &= \frac{(\alpha + \rho - \varepsilon - \psi)(B - \alpha S_1^*)}{S_1^*\beta(\alpha + \rho - \varepsilon - \psi) + \delta\beta\mu}, \\ T^* &= \frac{B\mu - \alpha\mu S_1^*}{S_1^*\beta(\alpha + \rho - \varepsilon - \psi) + \delta\beta\mu}, \\ R^* &= \frac{B\mu\rho - \rho\alpha\mu S_1^*}{S_1^*\alpha\beta(\alpha + \rho - \varepsilon - \psi) + \delta\beta\mu\alpha}. \end{aligned}$$

Let  $y = (I, T, R, S_1, S_2)^\mathcal{T}$ , then the fractional-order model (1.1) can be written as

$$y' = \mathcal{F}(y) - \mathcal{Z}(y),$$

where

$$\mathcal{F}(y) = \begin{bmatrix} \beta I(t)(S_1(t) + S_2(t)) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathcal{Z}(y) = \begin{bmatrix} \mu I(t) - \delta\beta T(t) + \alpha I(t) - \sigma I(t) \\ -\mu I(t) + \alpha T(t) + \rho T(t) - \varepsilon T(t) - \psi T(t) \\ \alpha R(t) - \rho T(t) \\ -b + \beta I(t)S_1(t) + \delta\beta T(t) + S_1(t) \\ -b + \beta I(t)S_2(t) + \delta\beta T(t) + S_2(t) \end{bmatrix}.$$

The Jacobian matrices of  $\mathcal{F}(y)$  and  $\mathcal{Z}(y)$  at  $P_0$  are, respectively,

$$F = \begin{bmatrix} \frac{2\beta B}{\alpha} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} A_{2 \times 2} & O_{2 \times 3} \\ C_{3 \times 2} & D_{3 \times 3} \end{bmatrix},$$

where

$$A_{2 \times 2} = \begin{bmatrix} \alpha + \mu - \sigma & -\delta\beta \\ -\mu & \alpha + \rho - \varepsilon - \psi \end{bmatrix}, \quad D_{3 \times 3} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix}, \quad C_{3 \times 2} = \begin{bmatrix} 0 & -\rho \\ \frac{b\beta}{\alpha} & \delta\beta \\ \frac{b\beta}{\alpha} & \delta\beta \end{bmatrix}.$$

Thus

$$V^{-1} = \begin{bmatrix} A_{2 \times 2}^{-1} & O_{2 \times 3} \\ E_{3 \times 2} & D_{3 \times 3}^{-1} \end{bmatrix}, \quad E_{3 \times 2} = -D_{3 \times 3}^{-1} C_{3 \times 2} A_{2 \times 2}^{-1}.$$

That is

$$V^{-1} = \begin{bmatrix} \frac{\alpha + \rho - \varepsilon - \psi}{(\alpha + \rho - \varepsilon - \psi)(\alpha + \mu - \sigma) - \delta\beta\mu} & \frac{\delta\beta}{(\alpha + \rho - \varepsilon - \psi)(\alpha + \mu - \sigma) - \delta\beta\mu} & 0 & 0 & 0 \\ \frac{\mu}{(\alpha + \rho - \varepsilon - \psi)(\alpha + \mu - \sigma) - \delta\beta\mu} & \frac{\alpha + \mu - \sigma}{(\alpha + \rho - \varepsilon - \psi)(\alpha + \mu - \sigma) - \delta\beta\mu} & 0 & 0 & 0 \\ \frac{\mu}{(\alpha + \rho - \varepsilon - \psi)(\alpha + \mu - \sigma) - \delta\beta\mu} & \frac{\alpha + \mu - \sigma}{(\alpha + \rho - \varepsilon - \psi)(\alpha + \mu - \sigma) - \delta\beta\mu} & \alpha^3 & 0 & 0 \\ \frac{\rho\mu\alpha^3\beta^2}{(\alpha + \rho - \varepsilon - \psi)(\alpha + \mu - \sigma) - \delta\beta\mu} & \frac{\rho\alpha^3\beta^2(\alpha + \mu - \sigma)}{(\alpha + \rho - \varepsilon - \psi)(\alpha + \mu - \sigma) - \delta\beta\mu} & 0 & \alpha^3 & 0 \\ \frac{\rho\mu\alpha^3\beta^2}{(\alpha + \rho - \varepsilon - \psi)(\alpha + \mu - \sigma) - \delta\beta\mu} & \frac{\rho\alpha^3\beta^2(\alpha + \mu - \sigma)}{(\alpha + \rho - \varepsilon - \psi)(\alpha + \mu - \sigma) - \delta\beta\mu} & 0 & 0 & \alpha^3 \end{bmatrix}.$$

The spectral radius of the matrix  $F.V^{-1}$  is given by

$$\rho(F.V^{-1}) = \frac{\delta\beta\mu}{(\alpha + \rho - \varepsilon - \psi)(\alpha + \mu - \sigma - \frac{2B\beta}{\alpha})}.$$

Then, the basic reproduction number  $\mathcal{R}_0$  is given by

$$\mathcal{R}_0 = \frac{\delta\beta\mu}{(\alpha + \rho - \varepsilon - \psi)(\alpha + \mu - \sigma - \frac{2B\beta}{\alpha})}. \quad (2.1)$$

### 3. SENSITIVITY STATISTICAL ANALYSIS

Sensitivity statistical analysis is used to evaluate the relative influence of several factors on a model's stability when data is unknown. The analysis can also determine which parameters are crucial. Using both local and global techniques, we calculate the sensitivity indices of the basic reproduction number,  $\mathcal{R}_0$ , to the parameters in the model. The normalized forward sensitivity index  $\mathcal{R}_0$  is used in the local sensitivity analysis. The sensitivity index of  $\mathcal{R}_0$  with respect to the parameters in our model are derived as follows:

$$\Gamma_v^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial v} \times \frac{v}{\mathcal{R}_0}$$

where  $v$  is a value from Table 2, and  $\mathcal{R}_0$  is derived from Eq (2.1).

It is easy to verify that

$$\begin{aligned} \frac{\partial \mathcal{R}_0}{\partial B} &= \frac{2\delta\beta^2}{\alpha(\alpha + \rho - \varepsilon - \psi) \left(\alpha + \mu - \sigma - \frac{2B\beta}{\alpha}\right)^2} > 0, \\ \frac{\partial \mathcal{R}_0}{\partial \beta} &= \frac{\delta\mu \left(\alpha + \mu - \sigma - \frac{2B\beta}{\alpha}\right) - (\delta\beta\mu) \left(-\frac{2B}{\alpha}\right)}{(\alpha + \rho - \varepsilon - \psi) \left(\alpha + \mu - \sigma - \frac{2B\beta}{\alpha}\right)^2} > 0, \\ \frac{\partial \mathcal{R}_0}{\partial \delta} &= \frac{\beta\mu}{(\alpha + \rho - \varepsilon - \psi) \left(\alpha + \mu - \sigma - \frac{2B\beta}{\alpha}\right)} < 0, \\ \frac{\partial \mathcal{R}_0}{\partial \alpha} &= \frac{-\delta\beta\mu \left[\left(\alpha + \mu - \sigma - \frac{2B\beta}{\alpha}\right) + (\alpha + \rho - \varepsilon - \psi) \left(1 + \frac{2B\beta}{\alpha^2}\right)\right]}{(\alpha + \rho - \varepsilon - \psi)^2 \left(\alpha + \mu - \sigma - \frac{2B\beta}{\alpha}\right)^2} < 0, \\ \frac{\partial \mathcal{R}_0}{\partial \mu} &= \frac{\delta\beta \left(\alpha - \sigma - \frac{2B\beta}{\alpha}\right)}{(\alpha + \rho - \varepsilon - \psi) \left(\alpha + \mu - \sigma - \frac{2B\beta}{\alpha}\right)^2} < 0, \\ \frac{\partial \mathcal{R}_0}{\partial \sigma} &= \frac{\delta\beta\mu}{(\alpha + \rho - \varepsilon - \psi) \left(\alpha + \mu - \sigma - \frac{2B\beta}{\alpha}\right)^2} > 0, \\ \frac{\partial \mathcal{R}_0}{\partial \rho} &= \frac{-\delta\beta\mu}{(\alpha + \rho - \varepsilon - \psi)^2 \left(\alpha + \mu - \sigma - \frac{2B\beta}{\alpha}\right)} > 0, \\ \frac{\partial \mathcal{R}_0}{\partial \varepsilon} &= \frac{\delta\beta\mu}{(\alpha + \rho - \varepsilon - \psi)^2 \left(\alpha + \mu - \sigma - \frac{2B\beta}{\alpha}\right)} < 0, \\ \frac{\partial \mathcal{R}_0}{\partial \psi} &= \frac{\delta\beta\mu}{(\alpha + \rho - \varepsilon - \psi)^2 \left(\alpha + \mu - \sigma - \frac{2B\beta}{\alpha}\right)} < 0. \end{aligned} \tag{3.1}$$

The sensitivity indices revealed the delicacies of variable  $\mathcal{R}_0$  to the model parameters. The positive (negative) index indicate that an increase in the parameter value leads to an increase (decrease) of  $\mathcal{R}_0$  value. The sensitivity index of each parameter in model (1.1) are depicted in Table 2.

We analyze the sensitivity of  $\mathcal{R}_0$  by substituting the parameter values into Eq (3.1) as follows:

$$\begin{aligned}\Gamma_B^{\mathcal{R}_0} &= \frac{\partial \mathcal{R}_0}{\partial B} \times \frac{B}{\mathcal{R}_0} = -18.6, \\ \Gamma_\rho^{\mathcal{R}_0} &= \frac{\partial \mathcal{R}_0}{\partial \rho} \times \frac{\rho}{\mathcal{R}_0} = -1.2, \\ \Gamma_\sigma^{\mathcal{R}_0} &= \frac{\partial \mathcal{R}_0}{\partial \sigma} \times \frac{\sigma}{\mathcal{R}_0} = -0.11, \\ \Gamma_\beta^{\mathcal{R}_0} &= \frac{\partial \mathcal{R}_0}{\partial \beta} \times \frac{\beta}{\mathcal{R}_0} = -0.0094, \\ \Gamma_\delta^{\mathcal{R}_0} &= \frac{\partial \mathcal{R}_0}{\partial \delta} \times \frac{\delta}{\mathcal{R}_0} = 0.18, \\ \Gamma_\varepsilon^{\mathcal{R}_0} &= \frac{\partial \mathcal{R}_0}{\partial \varepsilon} \times \frac{\varepsilon}{\mathcal{R}_0} = 0.40, \\ \Gamma_\mu^{\mathcal{R}_0} &= \frac{\partial \mathcal{R}_0}{\partial \mu} \times \frac{\mu}{\mathcal{R}_0} = 0.59, \\ \Gamma_\psi^{\mathcal{R}_0} &= \frac{\partial \mathcal{R}_0}{\partial \psi} \times \frac{\psi}{\mathcal{R}_0} = 0.80, \\ \Gamma_\alpha^{\mathcal{R}_0} &= \frac{\partial \mathcal{R}_0}{\partial \alpha} \times \frac{\alpha}{\mathcal{R}_0} = 4.2.\end{aligned}$$

Thus, one obtains

Parameters	Description	Sensitivity Index
$B$	0.3	-18.6
$\rho$	0.3	-1.2
$\sigma$	0.005	-0.11
$\beta$	0.35	-0.0094
$\delta$	0.3	0.18
$\varepsilon$	0.1	0.40
$\mu$	0.55	0.59
$\psi$	0.2	0.80
$\alpha$	0.25	4.2

Table 2: Parameters description for the novel (COVID-19) SITR model

It is noted from the sensitivity indices given in Table 2 that the value of  $\mathcal{R}_0$  increases when the parameter values  $\delta$ ,  $\varepsilon$ ,  $\mu$ ,  $\psi$ , and  $\alpha$  increase while other parameter values are kept fixed. This implies an increase in the endemicity of the disease since the indices have positive signs. On the other hand, when the parameter values  $B$ ,  $\rho$ ,  $\sigma$ , and  $\beta$  are decreased while the rest of the parameter values are kept fixed, the value

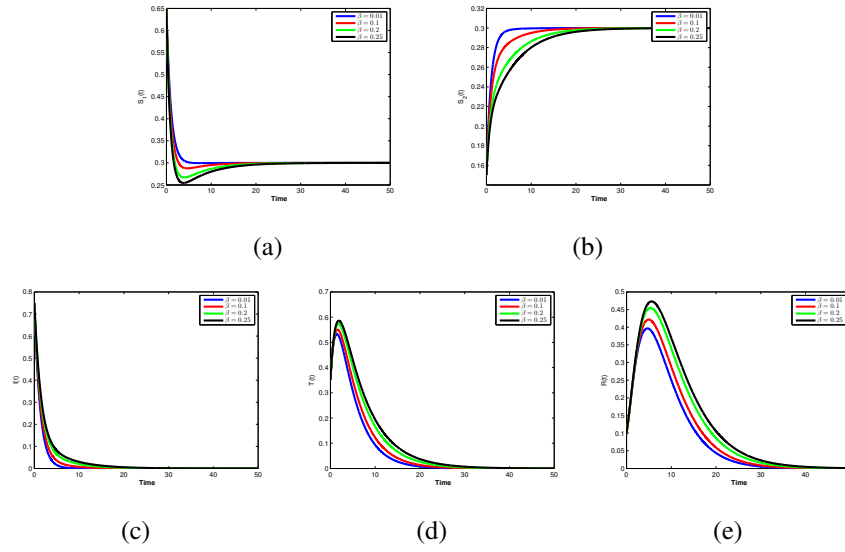


Figure 1: Solution profiles for  $S_1(t)$ ,  $S_2(t)$ ,  $I(t)$ ,  $T(t)$ , and  $R(t)$  with different  $\beta$  when  $\nu = 1$  and  $\mathcal{R}_0 = 0.4190$

of  $\mathcal{R}_0$  decreases. This shows a decrease in the disease endemicity because the indices have negative signs. The death rate  $\alpha$  and healthy food rate  $\psi$  are the most sensitive parameters. The recovery rate  $\mu$  and sleep rate  $\varepsilon$  are the other key parameters that are sensitive.

#### 4. NUMERICAL SIMULATION

We investigate the effects of the contact rate  $\beta$  in the COVID-19 spread dynamics. Figures 1 to 3 show the implications of the effects of different contact rates on the downward modulation of this virus. A smaller contact rate, similar to the effects of fractional order, significantly delays the peak and decreases the number of infected cases, as shown in Fig. 1 (c), Fig. 2 (c), and Fig. 3 (c). For the other fractional orders, the effect of contact rate on the dynamics is strong, see Figs. 1 to 3. In infected cases, the reduction of contact parameters leads to a drastic decrease, where a reduction in contact rates, with other parameters set, keeps  $\mathcal{R}_0$  under 1. Vaccines are, as at present, advised to reduce the rate of contact and the spread of CoV-19. It should be noted that when controlling the transmission rate with the lower fractional order  $\nu$ , there is a significant reduction in infected cases. The absence of infection outbreaks and a noticeable decrease in infectious waves were seen in our findings. Fig. 4 shows the simulation output of the COVID-19 model for  $\mathcal{R}_0 = 0.4190 < 1$ .

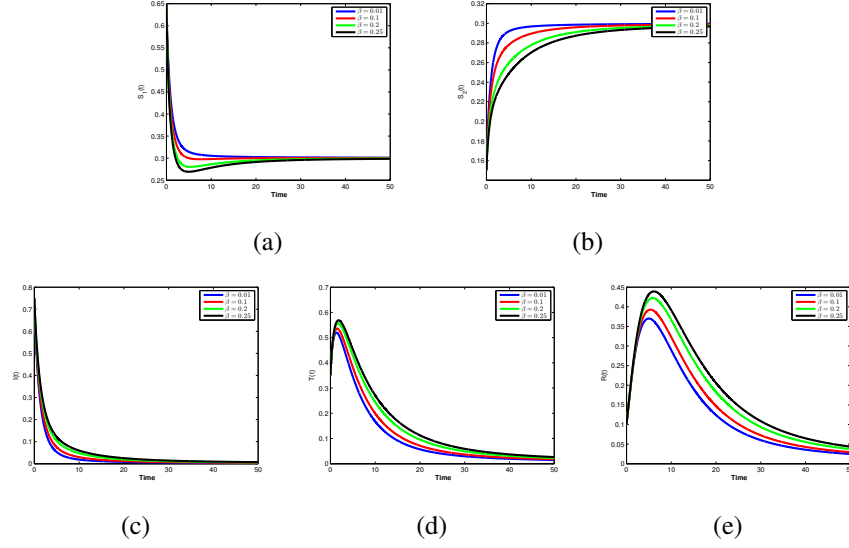


Figure 2: Solution profiles for  $S_1(t)$ ,  $S_2(t)$ ,  $I(t)$ ,  $T(t)$ , and  $R(t)$  with different  $\beta$  when  $\nu = 0.9$  and  $\mathcal{R}_0 = 0.4190$

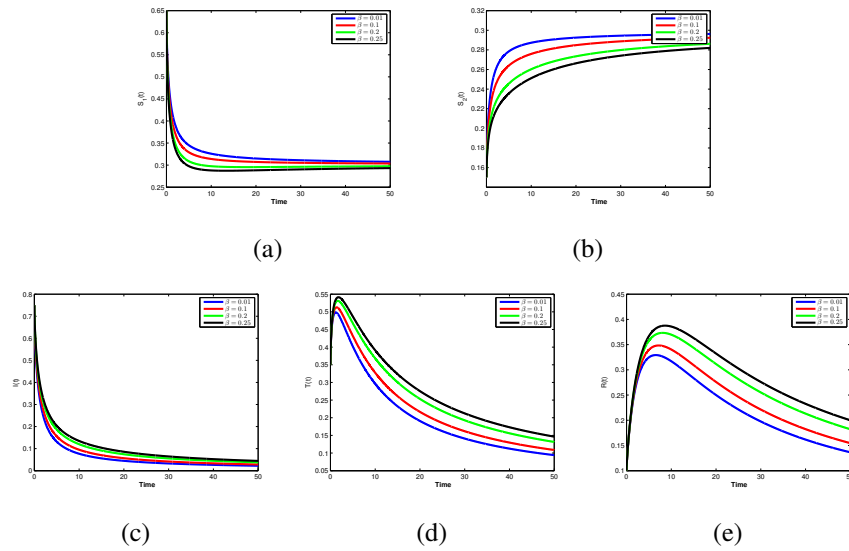
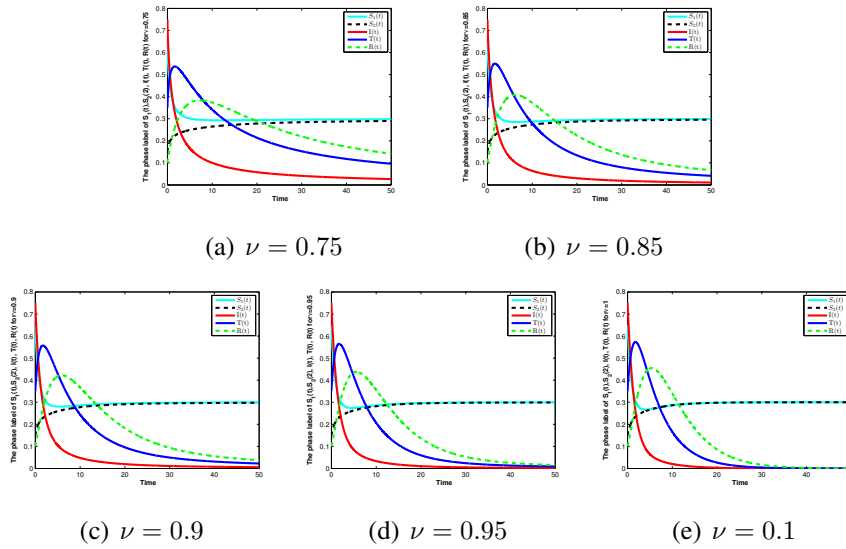


Figure 3: Solution profiles for  $S_1(t)$ ,  $S_2(t)$ ,  $I(t)$ ,  $T(t)$ , and  $R(t)$  with different  $\beta$  when  $\nu = 0.7$  and  $\mathcal{R}_0 = 0.4190$



Figure 4: Phase plot ( $S_1 - S_2 - I - T - R$ ) for  $\mathcal{R}_0 = 0.4190$ 

## 5. CONCLUSIONS

The Sitr fractal model, extended by the Caputo fractional derivative, is established in sense of four susceptible (S), infected (I), treatment (T), and recovered individual (R). The basic reproduction number determining whether the infection is extinguished was obtained. Sensitivity statistical analysis was carried out to see how changes in parameters affect the initial transmission of the Covid-19 disease. The next step is to perform a sensitivity analysis to determine which parameter has the greatest impact on disease endemicity. The parameters  $B$ ,  $\rho$ ,  $\sigma$ , and  $\beta$  are the most prevalent sensitivity indices towards the fundamental reproductive number, according to the data. To provide a realistic point of view, a numerical demonstration is presented via computer simulations using MATLAB.

### Availability of data and materials

The data in this work taken from the reference [12] and [13].

### Authors' contributions

All authors read and approved the final manuscript.

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