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Motion of Shock Wave through a Channel

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Abstract

An easy however suitable method is used to generate cylindrical converging shock waves. The shock dynamics is used to layout a curved wall profile of the check segment in a shock tube. When a planar shock wave propagates alongside the curved wall, the disturbances produced via way of means of the curved wall could constantly propagate alongside the shock floor and bend the shock wave. In the present paper there is analysis of Mach number with respect to various factors like μ and $[\Delta]$ including specific heat ratioγ, area of cross-section A of the used channel etc. The analysis has been done for the subsonic regime i.e., $M < 0.8$, transonic i.e., $0.8 < M < 1.2$, at $M=1$ i.e. at the speed of sound and supersonic regime, i.e., $1.2 < M < 5.0$.

Keywords: Mach number, Moving shock, Shock wave, Subsonic wave, Supersonic wave.

2010 AMS Subject Classification: 76L05

1. INTRODUCTION

The shock itself and the inflow behind it are perturbed, if a shock moves along a channel or a tube with a small area change. When're-reflected disturbances generated by non-uniformity behind the shock are neglected, inflow is called a freely propagating shock, as the shock surge isn't affected by there-reflected disturbances. Chisnell [1] and Whitham [2] have considered similar type of problem independently using different styles and have attained the relation between area of tube or channel and Mach number.

Numerous experimenters have studied the problem of re-reflected disturbances in the inflow behind the moving shock. Rosciszewski [3] has formulated the error involved in using CCW approximation and attained correction terms. Yousaf [4] has presented an exact expression of the strength of the disturbances over taking the shock. Milton [5] has attained a useful, simple relation between Mach number and area of the tube or channel.

The study of surge propagation in an admixture of gas and dust patches has entered great attention during the last several decades. There are numerous engineering operations for inflow of a medium that consists of suspense of powdered material or liquid driblets in a gas. Fine gas overflows have significance in engineering problems similar as inflow in rockets, nuclear-reactors, energy sprays, air pollution, etc. With the advancement of space technology, the dynamics of fluid flyspeck system has plant operations in extra-terrestrial field similar as lunar-ash- inflow and predictably in the studies of other globes. The dynamics of fine gas is modified from conventional gas dynamics by characterizing the temperature and haste of the gas and flyspeck independently. A single flyspeck that isn't in equilibrium with the gas inflow simply represents a poor ‘dick’ but if there are enough patches to form a significant bit of the mass of the admixture, their commerce with the gas affects the gas inflow, rather complicated overflows can thus develop as a result of the relaxation processes. As in the case of pure gas overflows, the rate at which diversions from equilibrium tend to be excluded may be fast or slow compared with the rate at which inflow changes take place. It's thus possible to consider ‘frozen’ inflow in which no relaxation processes take place, equilibrium flows for which relaxation is assumed to be infinitely presto, and intermediate non equilibrium flows.

Along with advances in colorful inflow fields mentioned over, some humorless sweats have been made in understanding the gusted of fine gas, starting with numerous simplifying hypothetical and variations. The paper by Marble [6] was an attempt in applying the ultramodern ways of fluid mechanics to the analysis of fine overflows. He has introduced numerous important generalities and parameters which can be served as strong via media in the development of the abecedarian equations of the admixture of gas and solid patches. Marble [7] give an expansive study of the overflows of the fine gas with illustration of shock conformation. Rudinger [8] has presented the thermodynamic parcels of shock swells, steady snoot inflow and general non steady one-dimensional inflow of the gas flyspeck admixture with colorful exemplifications depicting the significance of haste and temperature relaxations. Jena and Sharma [9] have studied the tone-analogous shocks in fine feasts. Following Whitham [2], Pandey and Verma [10] have bandied the conformation of shock down anon-uniform tube in two phase overflows.

The fine analysis of similar two phase inflow is vastly more delicate than that of pure gas overflows and one of the usual simplifying hypothetical is that the volume enthralled by the patches can be neglected. In numerous important cases, the flyspeck represents lower than one half of the mass of gas flyspeck admixture and the viscosity of the flyspeck material is further than thousand times larger than the gas viscosity. Under similar conditions the flyspeck volume bit is of order of 10^{-4} and supposition of a negligible flyspeck volume is also well satisfied. One more important consequence of this supposition is that equilibrium inflow of the admixture of patches with a perfect gas can be anatomized like inflow of perfect gas that has viscosity and specific heats of admixture. Carrier [11] was first to study the stir of shock surge in fine feasts. Colorful aspects of two- phase flows were studied by Soo [12], Kribe [13], Rudinger [14], Marble [15], Bailey [16], Kliegel [17], Gilbert [18], Kliegel [19].

At high gas densities (high pressure) or at high particle mass fragments, the flyspeck volume bit may come sufficiently large, so that it may be included into inflow analysis without introducing significant error. Since the patches may be considered as incompressible in comparison with the gas, the flyspeck volume bit enters into the introductory inflow equations as a fresh variable. The intriguing parcels of similar two phase flows are that indeed equilibrium flows cannot be treated as perfect gas overflows. There are numerous engineering problems in which dilute phase of gas patches is a good approximation of factual conditions. In similar cases due to the actuality of solid patches in the gas, parcels of admixture differ significantly from those of gas alone. Similar types of studies have multitudinous operations in underground explosion [20, 21].

In present paper Re-reflection effect on shock swells in two phase flows through a tube of variable cross section is considered when flyspeck volume bit appeared as a fresh variable [22]. Originally, Re-reflection goods on shock surge in a tube of variable cross section is attained and secondly, one dimensional area relation for anon-uniform, steady inflow ahead of a shock is attained and concluded that all the results are valid for the case when direction of the shock stir and the gas inflow ahead of the shock is same [23].

2. EQUATIONS OF MOTION

The differential equation for the motion of shock wave through tube given by Zhai et al. [24] can be written as:

$$\frac{2M\,dM}{(M^2-1)\,K(M)} + \frac{dA}{A} = 0 \tag{2.1}$$

$$\frac{1}{K(M)} = \left(1 + \frac{2}{\gamma+1}\,\frac{1-\mu^2}{\mu}\right)\frac{2\mu+1+M^{-2}}{2} \tag{2.2}$$

$$\mu = \sqrt{\frac{(\gamma-1)M^2+2}{2\gamma M^2-(\gamma-1)}} \tag{2.3}$$

Where M is the Mach number, A is the area of cross-section of the tube and γ is the specific ratio of heat of the used gas. The equation (2.1) takes the form:

$$\frac{2M}{(M^2-1)}\left(\frac{\gamma+1}{4} + \frac{1}{2\mu} - \frac{\mu}{2}\right)\left(2\mu + 1 + \frac{1}{M^2}\right)(1-\gamma+\gamma^2)\,dM + \frac{dA}{A} = 0 \tag{2.4}$$

$$[\Delta][(2\mu+1)\log(M^2-1)-2\log M+\log(M-1)+\log(M+1)]+\log A=\log C \quad (2.5)$$

Where;

$$[\Delta]=heat\ factor=(1-\gamma+\gamma^2)\left(\frac{\gamma+1}{4}+\frac{1}{2\mu}-\frac{\mu}{2}\right) \quad (2.6)$$

C=Constant and calculated by using the initial boundary conditions:

$$M=M_0,\ \mu=\mu_0,\ [\Delta]=[\Delta]_0 \quad (2.7)$$

$$[\Delta]_0\left[(2\mu_0+1)\log\left({M_0}^2-1\right)-2\log M_0+\log(M_0-1)+\log(M_0+1)\right]+\log A=\log C \quad (2.8)$$

$$\frac{(M^2-1)^{(2\mu+1)+[\Delta]}}{M^2}=\frac{C}{A} \quad (2.9)$$

$$\frac{(M^2-1)^{(2\mu+1)+[\Delta]}}{M^2}=\frac{1}{F(M)} \quad (2.10)$$

$$\Rightarrow F(M)\propto M \quad (2.11)$$

$$F(M)=\frac{A}{C}\Rightarrow F(M)\propto A \quad (2.12)$$

$$\Rightarrow M\propto A \quad (2.13)$$

3. MATHEMATICAL ANALYSIS

Table 2.1 (Values of the factors μ and $[\Delta]$ with respect to Mach number M)

S. No.	γ	M	A m^2	μ	$[\Delta]$
1	1.404	0.5	0.001256	7.0503	-4.4715
2	1.404	0.55	0.001256	4.5729	-1.5761
3	1.404	0.6	0.001256	3.5352	-1.0251
4	1.404	0.65	0.001256	2.7747	-0.9500
5	1.404	0.7	0.001256	2.3945	-0.3874
6	1.404	0.75	0.001256	1.8947	-0.1292
7	1.404	0.8	0.001256	1.6212	0.1548
8	1.404	0.9	0.001256	1.2441	0.3808
9	1.404	1.0	0.001256	1.0000	0.3510
10	1.404	1.1	0.001256	0.8313	0.7868
11	1.404	1.2	0.001256	0.7093	0.9512
12	1.404	1.3	0.001256	0.6179	1.1012
13	1.404	1.4	0.001256	0.5474	1.2407
14	1.404	1.5	0.001256	0.4918	1.3717

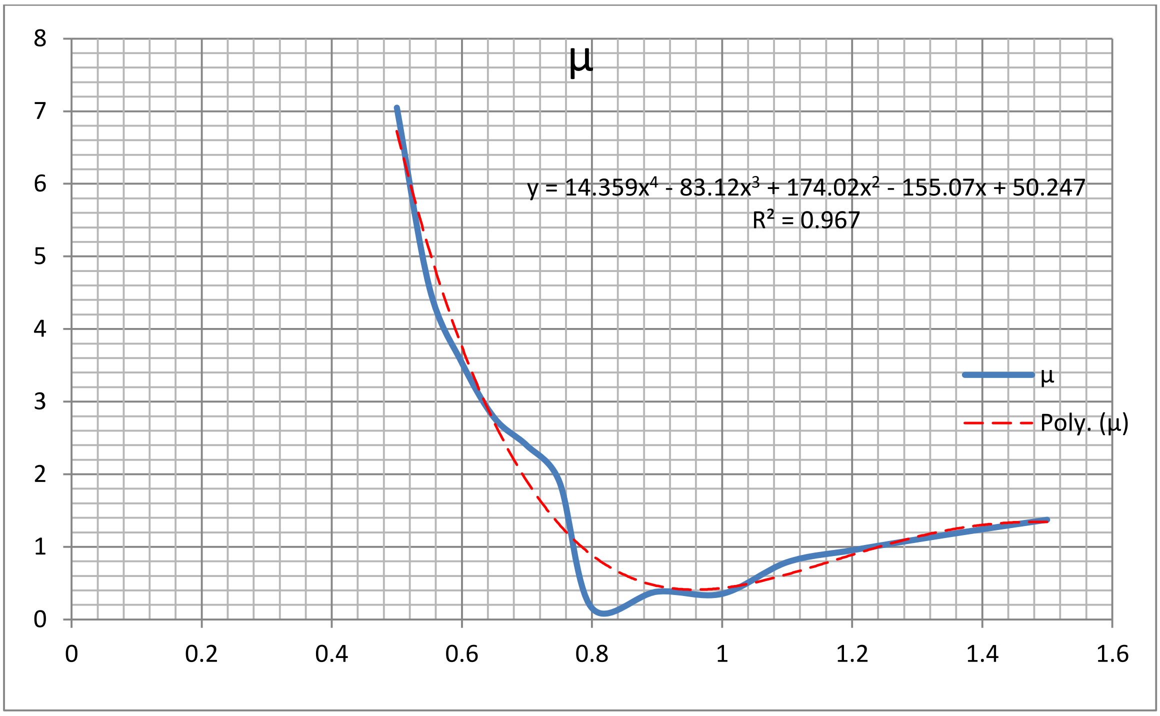

Figure 3.1 (Variation of factor μ with respect to Mach number)

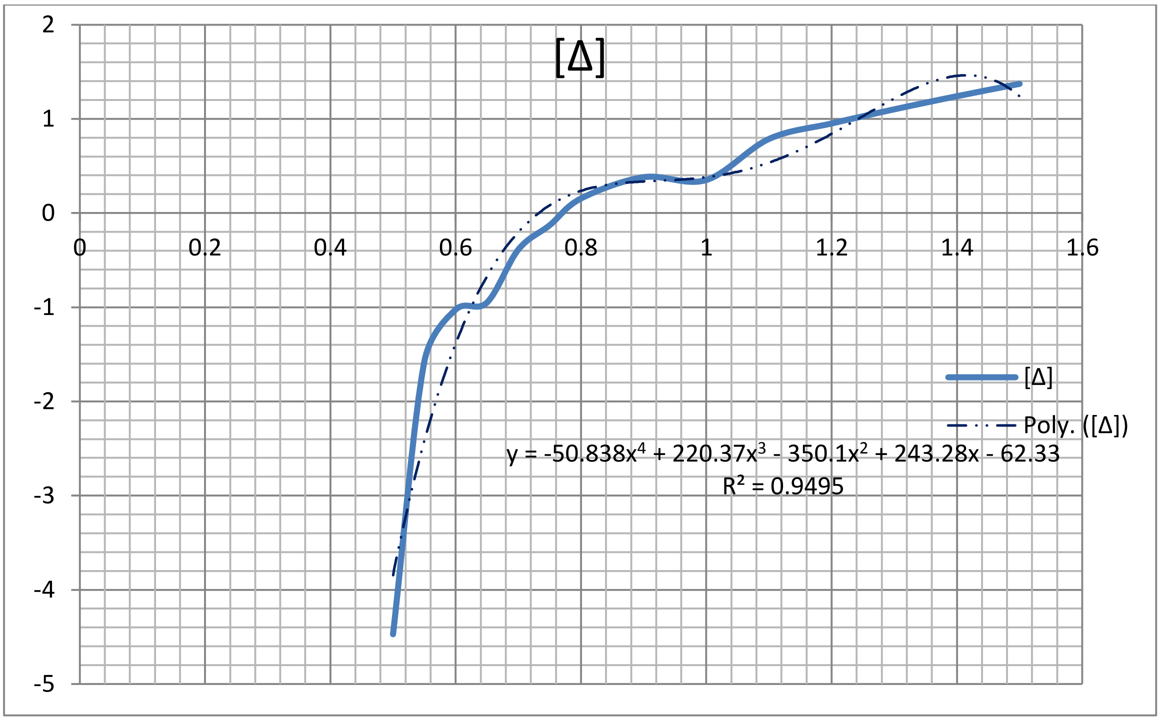

Figure 3.1 (Variation of heat factor $[\Delta]$ with respect to Mach number)

4. DISCUSSIONS

The calculated values of the factors μ and $[\Delta]$ with respect to Mach number M are shown in the Table 2.1 and figure 3.1 and 3.2 respectively. The figure 3.1, shows that for the subsonic regime i.e., $M < 0.8$, the factor μ decreases with M, whereas in the transonic and supersonic regime, it increases with M. The figure 3.2, shows that for the subsonic regime i.e., $M < 0.8$, the factor $[\Delta]$ increases with M, and in the transonic and supersonic regime, it also increases with M.

5. CONCLUSIONS

The calculated values of the factors μ and $[\Delta]$ with respect to Mach number M and figure 3.1 and 3.2, that shows for the subsonic regime i.e., $M < 0.8$, the factor μ decreases with M, whereas in the transonic and supersonic regime, it increases with M. It shows that for the subsonic regime i.e., $M < 0.8$, the factor $[\Delta]$ increases with M, and in the transonic and supersonic regime, it also increases with M. The equation (2.13) shows that the Mach number varies with the area of cross-section of the channel/ tube. The polynomial trend line equations are given by

$$\mu = 14.35\, M^4 - 83.12\, M^3 + 174\, M^2 - 155\, M + 50.24;\ R^2 = 0.967$$

$$[\Delta] = -50.83\, M^4 + 220.3\, M^3 - 350.1\, M^2 + 243.2\, M - 62.33;\ R^2 = 0.949$$

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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