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Effect of Catastrophe on Bulk Size Queueing Model

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Abstract

Consider a Markovian queue in which arrival of the units occur in batches according to Poisson process and the arriving units are served in batches with a fixed size and service times follow exponential distribution. The occurrence of catastrophe and vacation policies is employed to analyze the model. By applying probability generating functions the expected number of units in the queue, second factorial moments and variances when the server is busy and that is on vacation are mathematically derived. The numerical values are computed under certain conditions such as fixed or variable vacation, catastrophe and arrival rates and their respective curves are exhibited.

Key words: Markovian queue, bulk size, server vacation, catastrophe, mean queue sizes and variances.

1. INTRODUCTION

Queueing theory, a branch of applied probability deals with one of the most unwanted experiences of life, waiting. This subject tries to ease the real-life problems and is mostly applied in industries, medical field, telecommunications, network systems and so on. Bulk size rule, vacation policy, retrial technique, feedback rule are some of the techniques applied in different queueing models. The arrival or service pattern may follow Poisson process, Erlang distribution, hyper-exponential distribution, general distribution, phase-type distribution and etc.

Catastrophes occur at random, leading to extinction of all the customers and a brief inactivation of the service facility until the new arrival. Catastrophe may occur either from within the system or from another service system like a virus infecting the computer networks or environment and temperature affecting the movement of cockroaches and other insects (Jain and Kanethia, 2006). Queueing systems with catastrophes have been studied some years ago. The concept of losing customers due to catastrophic failures was first introduced by Gelenbe (Gelenbe, 1989). Notable models with catastrophe have been developed by many researchers (Chang et al., 2007; Chen and Renshaw, 1997; Di Crescenzo et al., 2003; Kumar and Soundararajan, 2002).

Any queueing system is performed by adopting any one or combination of queueing concepts which are mentioned above. In particular, vacation and catastrophe are utilized to construct the new queueing system. Many studies have described the transient solution or behaviour for queueing system with catastrophe, server failure, repair time, feedback and derived steady state probability, performance measures using probability generating function, continued fraction technique etc. (Chandrasekaran and Saravanarajan, 2012; Jain and Kumar, 2007; Kumar and Soundararajan, 2002; Kumar et al., 2007a, 2007b; Kumar and Madheswari, 2005; Sophia, 2016; Thangaraj and Vanitha, 2009; Vinodhini and Vidhya, 2014; Sundar Rajan et al., 2011; Balasubramanian et al., 2015)

Likewise, various queueing models with above concepts along with different vacation policies have been discussed by Ayyappan et al., 2013; Sundar Rajan et al., 2017; Jeyakumar and Gunasekaran, 2017; Parimala and Palaniammal, 2014. Also, the same system has been studied with time dependent solutions by Ayyappan et al., 2014; Sudhesh and Vaithiyanathan, 2015. Boudali and Economou, 2013 have discussed the effect of catastrophes on the strategic customer behaviour in queueing system and obtained customer equilibrium strategies for joining versus balking.

In this paper, a single server Markovian queue in which the units arrived in batches as well as served in batches with Bailey's (Norman T. J. Bailey, 1954) fixed size have been considered. Using probability generating functions, the mathematical expressions for mean queue sizes, factorial moments and variances are derived under the condition that the server is busy and that on vacation. The numerical values for the derived results are computed and their respective curves showed the behaviors of the system.

2. CONSTRUCTION OF THE MODEL

Consider a Markovian queue in which arrival of the units occur in batches according to Poisson process with parameter λ. Let a_i be the first order probability that 'i' units arrive in a batch during a small interval of time under the condition that, $0 \leq a_i \leq 1$ and $\sum_{i=1}^{n} a_i = 1$.

The arriving units are served in batches with a fixed size and service times follow exponential distribution with parameter μ. This system is governed by a single server. In addition to that, the concept of catastrophe and vacation policy is applied to perform this queueing system. The time periods of catastrophe and vacation follow exponential distributions with parameter η and γ respectively. It is assumed that, initially, there are k units in the system at the time of entry of the server and starts service immediately in a batch of size k.

Upon completion of service, when the server finds less than k units in the queue, then the server goes for a single or multiple vacations based on the size of the queue. If there are k or greater than k units in the queue, then the first k units only will be selected for batch service. Whenever a catastrophe occurs in the system, all the units available in the system will be completely destroyed and server goes for multiple vacations. If there are less than k units in the queue on the server's return from vacation, the server immediately leaves for another vacation and until the server finds k or more units in the queue.

3. FORMULATION OF BASIC EQUATIONS

Consider a Markov process {N(t), S(t)}: Here N(t) = n, (n = 0,1,2…..) is the number of units in the queue at time t and S(t) = 1,2 represents the status of the server mentioned as busy and on vacation at time t respectively.

$P_{n,j}(t)$ is the probability that there are 'n' units in the queue at time t when the server is busy or that on vacation according to j = 1,2.

Based on the above assumptions, the differential-difference equations under transient conditions are framed.

$$P'_{0,1}(t) = -(\lambda+\mu+\eta)P_{0,1}(t) + \mu P_{k,1}(t) + \gamma P_{k,2}(t), \qquad n = 0 \tag{1}$$

$$P'_{n,1}(t) = -(\lambda+\mu+\eta)P_{n,1}(t) + \lambda \sum_{i=1}^{n} a_i P_{n-i,1}(t) + \mu P_{n+k,1}(t) + \gamma P_{n+k,2}(t), \qquad n \geq 1 \tag{2}$$

$$P'_{0,2}(t) = -(\lambda+\eta)P_{0,2}(t) + \mu P_{0,1}(t) + \eta, \qquad n = 0 \tag{3}$$

$$P'_{n,2}(t) = -(\lambda+\eta)P_{n,2}(t) + \lambda \sum_{i=1}^{n} a_i P_{n-i,2}(t) + \mu P_{n,1}(t), \qquad n = 1,2,\dots,\ k-1 \tag{4}$$

$$P'_{n,2}(t) = -(\lambda+\eta+\gamma)P_{n,2}(t) + \lambda \sum_{i=1}^{n} a_i P_{n-i,2}(t), \qquad n \geq k \tag{5}$$

The above stated transient state equations (1) to (5) are transformed into Steady-State equations by assuming $\lim_{t\to\infty} P_{n,i}(t) = P_{n,i}$ and $\lim_{t\to\infty} P'_{n,1}(t) = 0$ and the reduced equations are

$$(\lambda+\mu+\eta)P_{0,1} = \mu P_{k,1} + \gamma P_{k,2}, \qquad n = 0 \tag{6}$$

$$(\lambda+\mu+\eta)P_{n,1} = \lambda \sum_{i=1}^{n} a_i P_{n-i,1} + \mu P_{n+k,1} + \gamma P_{n+k,2}, \qquad n \geq 1 \tag{7}$$

$$(\lambda+\eta)P_{0,2} = \mu P_{0,1} + \eta, \qquad n = 0 \tag{8}$$

$$(\lambda+\eta)P_{n,2} = \lambda\sum_{i=1}^{n} a_i P_{n-i,2} + \mu P_{n,1}\,, \qquad n = 1,2,\dots,\quad k-1 \tag{9}$$

$$(\lambda+\eta+\gamma)P_{n,2} = \lambda\sum_{i=1}^{n} a_i P_{n-i,2}\,, \qquad n \geq k \tag{10}$$

4. PROBABILITY GENERATING FUNCTIONS

The probability generating functions when the server is busy and that on vacation are respectively denoted as G(Z) and H(Z). Also, the probability generating function of the arrival process is A(Z).

$$\left.\begin{aligned} \text{i.e.,}\quad & G(Z) = \sum_{n=0}^{\infty} P_{n,1} z^n \\ & H(Z) = \sum_{n=0}^{\infty} P_{n,2} z^n \\ \text{and}\quad & A(Z) = \sum_{i=1}^{n} a_i z^i \end{aligned}\right\} \tag{11}$$

Multiply the equation (7) by z^n and summing over n = 1,2,3…….∞ along with the equation (6) and apply the expression (11) and get,

$$[(\lambda+\mu+\eta)z^k - \lambda A(Z)z^k - \mu]G(Z) = \gamma H(Z) - \mu\sum_{n=0}^{k-1} P_{n,1} z^n - \gamma\sum_{n=0}^{k-1} P_{n,2} z^n \tag{12}$$

Similarly apply the same process in the equations (8), (9) and (10) and utilize the expressions given in (11) and get,

$$[(\lambda+\eta+\gamma) - \lambda A(Z)]H(Z) = \mu\sum_{n=0}^{k-1} P_{n,1} z^n + \gamma\sum_{n=0}^{k-1} P_{n,2} z^n + \eta \tag{13}$$

The equation (13) is re-written as

$$H(Z) = \frac{B(Z)+\eta}{\lambda+\eta+\gamma-\lambda A(Z)} \tag{14}$$

Where,

$$B(Z) = \mu\sum_{n=0}^{k-1} P_{n,1} z^n + \gamma\sum_{n=0}^{k-1} P_{n,2} z^n$$

In this juncture, substitute the expression (14) in the equation (12) which provide

$$G(Z) = \frac{(\lambda+\eta-\lambda A(Z)B(Z)-\gamma\eta}{[\lambda+\eta+\gamma-\lambda A(Z)]\{\lambda A(Z)z^k+\mu-(\lambda+\eta+\mu)z^k\}} \tag{15}$$

The expressions (14) and (15) are the required probability generating functions respectively when the server is on vacation and the server is busy.

5. PERFORMANCE MEASURES

The mean and the variance of the number of units in the queue are obtained under the condition that the server is busy and that on vacation. For this purpose, differentiate *G*(*Z*) and *H*(*Z*) and letting Z→1. But the expressions lead to in-determinant form. Hence, apply L' Hospital's rule and get the required results.

The expected number of units in the queue when the server is busy is given by

$$G'(1) = \frac{k(\gamma+\eta)(\mu+\eta)B(1)+\lambda\eta A'(1)B(1)-\eta(\gamma+\eta)B'(1)}{\eta^2(\gamma+\eta)^2} \qquad (16)$$

Similarly, the expected number of units in the queue when the server is on vacation is obtained as

$$H'(1) = \frac{(\gamma+\eta)B'(1)+\lambda A'(1)\{\eta+B(1)\}}{(\gamma+\eta)^2} \qquad (17)$$

Now, the respective second factorial moments are derived.

$$G''(1) = \frac{\alpha}{\beta} \qquad (18)$$

Where,

$$\alpha = \eta^2(\gamma+\eta)^2\ [\eta\ (\gamma+\eta)\{2\lambda A'(1)B'(1) - \eta\ B''(1) + \lambda\ B(1)A''(1)\} + \lambda\ \eta\ \xi_2 A'(1)$$
$$+(\gamma+\eta)\xi_1\xi_2 + \big(\gamma\ \eta - \eta\ B(1)\big)[-\lambda\ \xi_1\ A'(1) + \lambda\ \eta\ A''(1)$$
$$+(\gamma+\eta)\big\{\lambda\ \big(k\ (k-1) + 2kA'(1) + A''(1)\big) - k(k-1)(\lambda+\mu+\eta)\big\}$$
$$-\lambda\ \xi_1 A'(1)] + \xi_2\xi_3\ ] + 2\ (\gamma+\eta)\ \eta^2\ \xi_2\ [-\xi_3(\gamma+\eta) + \big(\gamma - B(1)\big)\ \xi_3]$$

$$\beta = \eta^4(\gamma+\eta)^4$$

$$\xi_1 = \lambda A'(1) - (\mu+\eta)k$$

$$\xi_2 = \lambda B'(1) - \lambda B(1)A'(1) \quad \text{and}$$

$$\xi_3 = \lambda\eta A'(1) + (\gamma+\eta)\xi_1$$

$$H''(1) = (\gamma+\eta)^{-2}\big[(\gamma+\eta)B''(1) + \lambda A''(1)\big(\eta + B(1)\big)\big] + 2\lambda(\gamma+\eta)^{-3}A'(1)$$
$$\{(\gamma+\eta)B'(1) + \lambda A'(1)(\eta + B(1)\} \qquad (19)$$

For the purpose of estimating variances, construct a relation by using first and second factorial moments as

$$V\ = G''(1) + G'(1) - (G'(1))^2 \qquad (20)$$

By using the equations (16), (17), (18) and (19) in equation (20), the required variances of the number of units in the queue when the server is busy and that is on vacation are respectively obtained as

$$V_1 = 1/\beta\ [\ \alpha + \{\ k\ (\gamma+\eta)(\ \mu+\eta\ )B(1) + \lambda\ \eta\ A'(1)B(1) - \eta\ (\gamma+\eta)B'(1)\}$$
$$\sqrt{\beta} - k\ (\gamma+\eta)(\ \mu+\eta\ )B(1) - \lambda\ \eta\ A'(1)B(1) + \eta\ (\gamma+\eta)B'(1)\}] \qquad (21)$$

and,

$$V_2 = 1/(\ \gamma+\eta\ )^4\ \ [\ (\ \gamma+\eta\ )^2\{\ (\gamma+\eta)B''(1) + \lambda\ A''(1)\big(\ \eta + B(1)\big)\}$$
$$+\xi_4\ \{\ 2\ \lambda\ \ A'(1)\ (\gamma+\eta\ ) + (\ \gamma+\eta\ )^2\ \ - \xi_4\}] \qquad (22)$$

Where,

$$\xi_4 = (\ \gamma+\eta\ )\ B'(1) + \lambda\ A'(1)(\ \eta + B\ (1))$$

6. NUMERICAL ILLUSTRATIONS

The expected number of units in the queue, when the server is busy (ie., eqn.16) and that on vacation (ie., eqn.17), are computed numerically for the different values of η, λ and γ with the fixed values for μ, k, $A'(1)$, $B(1)$, and $B'(1)$ and presented in the following Tables (Tables 1-10) and graphs (Figure 1-5).

Let us consider, $k = 12$, $\mu = 15$, $A'(1) = 0.5$, $B(1) = 0.02$, $B'(1) = 0.04$ for the values of η = 0.5, 1.0, 2.0 , 3.5, 4.8, 7.5, γ = 0.3, 0.5, 0.8, 1.2, 1.7, and λ = 1, 2, 3, 4, 5, 6, 7, 8, 9 , 10, 15, 20, 25, 30, 35, 40, 45,50.

Table 1: Expected number of units in the queue G'(1) when the server is busy for different values of λ and η for fixed value of $\gamma = 0.3$

λ	*G'(1)*					
	$\gamma = 0.3$					
	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	$\eta = 3.5$	$\eta = 4.8$	$\eta = 7.5$
1	18.5313	2.9290	0.4357	0.0926	0.0389	0.0116
2	18.5625	2.9349	0.4367	0.0928	0.0390	0.0117
3	18.5938	2.9408	0.4376	0.0930	0.0390	0.0117
4	18.6250	2.9467	0.4386	0.0932	0.0391	0.0117
5	18.6563	2.9527	0.4395	0.0934	0.0392	0.0117
6	18.6875	2.9586	0.4405	0.0936	0.0393	0.0118
7	18.7188	2.9645	0.4414	0.0938	0.0394	0.0118
8	18.7500	2.9704	0.4423	0.0940	0.0394	0.0118
9	18.7813	2.9763	0.4433	0.0942	0.0395	0.0118
10	18.8125	2.9822	0.4442	0.0944	0.0396	0.0118
15	18.9688	3.0118	0.4490	0.0953	0.0400	0.0120
20	19.1250	3.0414	0.4537	0.0963	0.0404	0.0121
25	19.2813	3.0710	0.4584	0.0973	0.0408	0.0122
30	19.4375	3.1006	0.4631	0.0983	0.0412	0.0123
35	19.5938	3.1302	0.4679	0.0993	0.0416	0.0124
40	19.7500	3.1598	0.4726	0.1003	0.0420	0.0125
45	19.9063	3.1893	0.4773	0.1013	0.0424	0.0126
50	20.0625	3.2189	0.4820	0.1023	0.0428	0.0127

Table 2: Expected number of units in the queue H'(1) when the server is on vacation for different values of λ and η for fixed value of γ = 0.3

λ	*H'(1)*					
	γ = 0.3					
	η = 0.5	η = 1	η = 2	η = 3.5	η = 4.8	η = 7.5
1	0.4563	0.3325	0.2083	0.1324	0.1005	0.0669
2	0.8625	0.6343	0.3992	0.2543	0.1932	0.1287
3	1.2688	0.9361	0.5902	0.3762	0.2858	0.1905
4	1.6750	1.2379	0.7811	0.4981	0.3785	0.2523
5	2.0813	1.5396	0.9720	0.6199	0.4711	0.3141
6	2.4875	1.8414	1.1629	0.7418	0.5638	0.3759
7	2.8938	2.1432	1.3539	0.8637	0.6564	0.4377
8	3.3000	2.4450	1.5448	0.9856	0.7491	0.4995
9	3.7063	2.7467	1.7357	1.1075	0.8418	0.5613
10	4.1125	3.0485	1.9267	1.2294	0.9344	0.6231
15	6.1438	4.5574	2.8813	1.8388	1.3977	0.9321
20	8.1750	6.0663	3.8359	2.4482	1.8610	1.2412
25	10.2063	7.5751	4.7905	3.0576	2.3243	1.5502
30	12.2375	9.0840	5.7452	3.6670	2.7875	1.8592
35	14.2688	10.5929	6.6998	4.2765	3.2508	2.1682
40	16.3000	12.1018	7.6544	4.8859	3.7141	2.4772
45	18.3313	13.6107	8.6091	5.4953	4.1773	2.7862
50	20.3625	15.1195	9.5637	6.1047	4.6407	3.0952

Table 3: Expected number of units in the queue G'(1) when the server is busy for different values of λ and η for fixed value of γ = 0.5

λ	*G'(1)*					
	γ = 0.5					
	η = 0.5	η = 1	η = 2	η = 3.5	η = 4.8	η = 7.5
1	14.8200	2.5378	0.4008	0.0879	0.0374	0.0114
2	14.8400	2.5422	0.4016	0.0881	0.0375	0.0114
3	14.8600	2.5467	0.4024	0.0883	0.0376	0.0114
4	14.8800	2.5511	0.4032	0.0885	0.0376	0.0114
5	14.9000	2.5556	0.4040	0.0886	0.0377	0.0114
6	14.9200	2.5600	0.4048	0.0888	0.0378	0.0115
7	14.9400	2.5644	0.4056	0.0890	0.0379	0.0115
8	14.9600	2.5689	0.4064	0.0892	0.0379	0.0115
9	14.9800	2.5733	0.4072	0.0894	0.0380	0.0115
10	15.0000	2.5778	0.4080	0.0895	0.0381	0.0115
15	15.1000	2.6000	0.4120	0.0904	0.0385	0.0116
20	15.2000	2.6222	0.4160	0.0913	0.0388	0.0118
25	15.3000	2.6444	0.4200	0.0922	0.0392	0.0119
30	15.4000	2.6667	0.4240	0.0931	0.0396	0.0120
35	15.5000	2.6889	0.4280	0.0940	0.0399	0.0121
40	15.6000	2.7111	0.4320	0.0949	0.0403	0.0122
45	15.7000	2.7333	0.4360	0.0958	0.0407	0.0123
50	15.8000	2.7556	0.4400	0.0967	0.0411	0.0124

Table 4: Expected number of units in the queue H'(1) when the server is on vacation for different values of λ and η for fixed value of γ = 0.5

λ	*H'(1)*					
	γ = 0.5					
	η = 0.5	η = 1	η = 2	η = 3.5	η = 4.8	η = 7.5
1	0.3000	0.2533	0.1776	0.1200	0.0933	0.0638
2	0.5600	0.4800	0.3392	0.2300	0.1791	0.1225
3	0.8200	0.7067	0.5008	0.3400	0.2649	0.1813
4	1.0800	0.9333	0.6624	0.4500	0.3507	0.2400
5	1.3400	1.1600	0.8240	0.5600	0.4365	0.2988
6	1.6000	1.3867	0.9856	0.6700	0.5223	0.3575
7	1.8600	1.6133	1.1472	0.7800	0.6081	0.4163
8	2.1200	1.8400	1.3088	0.8900	0.6939	0.4750
9	2.3800	2.0667	1.4704	1.0000	0.7797	0.5334
10	2.6400	2.2933	1.6320	1.1100	0.8655	0.5925
15	3.9400	3.4267	2.4400	1.6600	1.2945	0.8863
20	5.2400	4.5600	3.2480	2.2100	1.7235	1.1800
25	6.5400	5.6933	4.0560	2.7600	2.1524	1.4738
30	7.8400	6.8267	4.8640	3.3100	2.5814	1.7675
35	9.1400	7.9600	5.6720	3.8600	3.0104	2.0613
40	10.4400	9.0933	6.4800	4.4100	3.4394	2.3550
45	11.7400	10.2267	7.2880	4.9600	3.8684	2.6488
50	13.0400	11.3600	8.0960	5.5100	4.2973	2.9425

Table 5: Expected number of units in the queue G'(1) when the server is busy for different values of λ and η for fixed value of $\gamma = 0.8$

λ	*G'(1)*					
	$\gamma = 0.8$					
	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	$\eta = 3.5$	$\eta = 4.8$	$\eta = 7.5$
1	11.3964	2.1142	0.3578	0.0818	0.0354	0.0109
2	11.4083	2.1173	0.3584	0.0819	0.0355	0.0110
3	11.4201	2.1204	0.3591	0.0821	0.0355	0.0110
4	11.4320	2.1235	0.3597	0.0823	0.0356	0.0110
5	11.4438	2.1265	0.3603	0.0824	0.0357	0.0110
6	11.4556	2.1296	0.3610	0.0826	0.0357	0.0110
7	11.4675	2.1327	0.3616	0.0827	0.0358	0.0111
8	11.4793	2.1358	0.3622	0.0829	0.0359	0.0111
9	11.4911	2.1389	0.3629	0.0830	0.0359	0.0111
10	11.5030	2.1420	0.3635	0.0832	0.0360	0.0111
15	11.5621	2.1574	0.3667	0.0840	0.0363	0.0112
20	11.6213	2.1728	0.3699	0.0847	0.0367	0.0113
25	11.6805	2.1883	0.3731	0.0855	0.0370	0.0114
30	11.7396	2.2037	0.3763	0.0863	0.0373	0.0115
35	11.7988	2.2191	0.3795	0.0870	0.0377	0.0116
40	11.8580	2.2346	0.3827	0.0878	0.0380	0.0117
45	11.9172	2.2500	0.3858	0.0886	0.0383	0.0118
50	11.9763	2.2654	0.3890	0.0894	0.0387	0.0119

Table 6: Expected number of units in the queue H'(1) when the server is on vacation for different values of λ and η for fixed value of γ = 0.8

λ	*H'(1)*					
	γ = 0.8					
	η = 0.5	η = 1	η = 2	η = 3.5	η = 4.8	η = 7.5
1	0.1846	0.1796	0.1431	0.1045	0.0840	0.0594
2	0.3385	0.3370	0.2719	0.1997	0.1608	0.1140
3	0.4923	0.4944	0.4008	0.2949	0.2377	0.1686
4	0.6462	0.6519	0.5296	0.3900	0.3145	0.2231
5	0.8000	0.8093	0.6584	0.4852	0.3914	0.2777
6	0.9538	0.9667	0.7872	0.5804	0.4682	0.3323
7	1.1077	1.1241	0.9161	0.6756	0.5451	0.3869
8	1.2615	1.2815	1.0449	0.7708	0.6219	0.4415
9	1.4154	1.4389	1.1737	0.8660	0.6988	0.4960
10	1.5692	1.5963	1.3026	0.9612	0.7756	0.5506
15	2.3385	2.3833	1.9467	1.4371	1.1599	0.8235
20	3.1077	3.1704	2.5908	1.9130	1.5441	1.0964
25	3.8769	3.9574	3.2349	2.3890	1.9284	1.3693
30	4.6462	4.7444	3.8791	2.8649	2.3120	1.6422
35	5.4154	5.5315	4.5232	3.3408	2.6969	1.9151
40	6.1846	6.3185	5.1673	3.8168	3.0811	2.1880
45	6.9538	7.1050	5.8115	4.2927	3.4653	2.4609
50	7.7231	7.8926	6.4556	4.7686	3.8496	2.7338

Table 7: Expected number of units in the queue G’(1) when the server is busy for different values of λ and η for fixed value of γ = 1.2

λ	*G'(1)*					
	γ = 1.2					
	η = 0.5	η = 1	η = 2	η = 3.5	η = 4.8	η = 7.5
1	8.7128	1.7293	0.3130	0.0748	0.0330	0.0104
2	8.7197	1.7314	0.3135	0.0749	0.0331	0.0105
3	8.7266	1.7335	0.3140	0.0751	0.0332	0.0105
4	8.7336	1.7355	0.3145	0.0752	0.0332	0.0105
5	8.7405	1.7376	0.3149	0.0753	0.0333	0.0105
6	8.7474	1.7397	0.3154	0.0755	0.0333	0.0105
7	8.7543	1.7417	0.3159	0.0756	0.0334	0.0105
8	8.7612	1.7438	0.3164	0.0757	0.0334	0.0106
9	8.7682	1.7459	0.3169	0.0758	0.0335	0.0106
10	8.7751	1.7479	0.3174	0.0760	0.0336	0.0106
15	8.8097	1.7583	0.3198	0.0766	0.0339	0.0107
20	8.8442	1.7686	0.3223	0.0773	0.0341	0.0108
25	8.8789	1.7789	0.3247	0.0779	0.0344	0.0109
30	8.9135	1.7893	0.3271	0.0786	0.0347	0.0109
35	8.9481	1.7996	0.3296	0.0792	0.0350	0.0110
40	8.9827	1.8099	0.3320	0.0799	0.0353	0.0111
45	9.0173	1.8202	0.3345	0.0805	0.0356	0.0112
50	9.0519	1.8306	0.3369	0.0812	0.0359	0.0113

Table 8: Expected number of units in the queue H'(1) when the server is on vacation for different values of λ and η for fixed value of γ = 1.2

λ	*H'(1)*					
	γ = 1.2					
	η = 0.5	η = 1	η = 2	η = 3.5	η = 4.8	η = 7.5
1	0.1135	0.1236	0.1111	0.0882	0.0730	0.0543
2	0.2035	0.2289	0.2098	0.1679	0.1406	0.1040
3	0.2934	0.3343	0.3084	0.2475	0.2075	0.1536
4	0.3834	0.4397	0.4070	0.3272	0.2744	0.2033
5	0.4734	0.5450	0.5057	0.4069	0.3414	0.2530
6	0.5633	0.6504	0.6043	0.4866	0.4083	0.3027
7	0.6533	0.7558	0.7029	0.5662	0.4753	0.3523
8	0.7433	0.8612	0.8016	0.6459	0.5422	0.4020
9	0.8332	0.9665	0.9002	0.7256	0.6292	0.4517
10	0.9232	1.0719	0.9988	0.8053	0.6761	0.5014
15	1.3730	1.5988	1.4920	1.2036	1.0108	0.7497
20	1.8228	2.1256	1.9852	1.6020	1.3450	0.9981
25	2.2727	2.6525	2.4783	2.0004	1.6803	1.2465
30	2.7225	3.1793	2.9715	2.3987	2.0150	1.4949
35	3.1723	3.7062	3.4646	2.7971	2.3497	1.7433
40	3.6221	4.2331	3.9578	3.1955	2.6844	1.9917
45	4.0720	4.7599	4.4510	3.5938	3.0192	2.2400
50	4.5218	5.2868	4.9441	3.9922	3.3539	2.4884

Table 9: Expected number of units in the queue G'(1) when the server is busy for different values of λ and η for fixed value of $\gamma = 1.7$

λ	*G'(1)*					
	$\gamma = 1.7$					
	$\eta = 0.5$	$\eta = 1$	$\eta = 2$	$\eta = 3.5$	$\eta = 4.8$	$\eta = 7.5$
1	6.7314	1.4088	0.2706	0.0676	0.0305	0.0099
2	6.7355	1.4102	0.2710	0.0677	0.0305	0.0099
3	6.7397	1.4115	0.2714	0.0678	0.0306	0.0099
4	6.7438	1.4129	0.2717	0.0679	0.0306	0.0099
5	6.7479	1.4143	0.2721	0.0680	0.0307	0.0099
6	6.7521	1.4156	0.2725	0.0681	0.0307	0.0099
7	6.7562	1.4170	0.2728	0.0682	0.0308	0.0100
8	6.7603	1.4184	0.2732	0.0683	0.0308	0.0100
9	6.7645	1.4198	0.2736	0.0685	0.0309	0.0100
10	6.7686	1.4211	0.2739	0.0686	0.0309	0.0100
15	6.7893	1.4280	0.2757	0.0691	0.0312	0.0101
20	6.8099	1.4348	0.2776	0.0696	0.0314	0.0102
25	6.8306	1.4417	0.2794	0.0701	0.0317	0.0102
30	6.8512	1.4486	0.2812	0.0707	0.0319	0.0103
35	6.8719	1.4554	0.2831	0.0712	0.0322	0.0104
40	6.8926	1.4623	0.2849	0.0717	0.0324	0.0105
45	6.9132	1.4691	0.2867	0.0723	0.0327	0.0106
50	6.9339	1.4760	0.2885	0.0728	0.0329	0.0106

Table 10: Expected number of units in the queue H'(1) when the server is on vacation for different values of λ and η for fixed value of γ = 1.7

λ	*H'(1)*					
	γ = 1.7					
	η = 0.5	η = 1	η = 2	η = 3.5	η = 4.8	η = 7.5
1	0.0719	0.0848	0.0846	0.0728	0.0632	0.0488
2	0.1256	0.1547	0.1584	0.1379	0.1202	0.0932
3	0.1793	0.2247	0.2321	0.2030	0.1773	0.1376
4	0.2331	0.2947	0.3059	0.2680	0.2343	0.1820
5	0.2868	0.3646	0.3797	0.3331	0.2914	0.2265
6	0.3405	0.4346	0.4535	0.3982	0.3484	0.2709
7	0.3942	0.5045	0.5272	0.4633	0.4054	0.3153
8	0.4479	0.5745	0.6010	0.5284	0.4625	0.3597
9	0.5017	0.6444	0.6748	0.5935	0.5195	0.4042
10	0.5554	0.7144	0.7486	0.6586	0.5767	0.4486
15	0.8240	1.0642	1.1175	0.9840	0.8618	0.6707
20	1.0926	1.4140	1.4863	1.3095	1.1470	0.8928
25	1.3612	1.7638	1.8552	1.6349	1.4322	1.1149
30	1.6298	2.1136	2.2241	1.9604	1.7174	.1.3371
35	1.8983	2.4634	2.5930	2.2858	2.0026	1.5592
40	2.1669	2.8132	2.9619	2.6112	2.2878	1.7813
45	2.4355	3.1630	3.3308	2.9367	2.5730	2.0034
50	2.7041	3.5128	3.6993	3.2621	2.8582	2.2255

Figure 1: Expected number of units in the queue when $\gamma = 0.3$

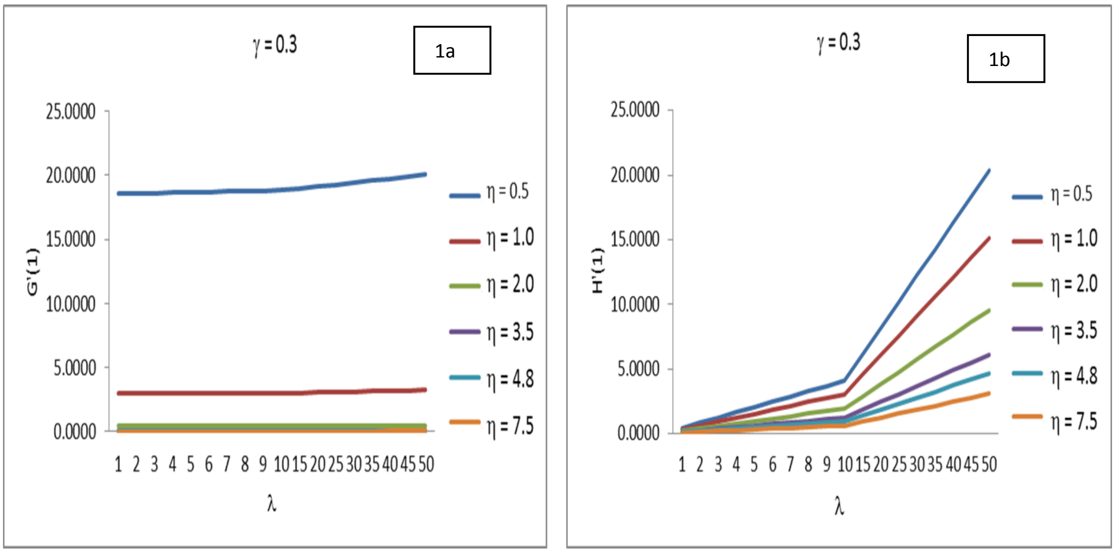

Figure 1a: Expected number of units in the queue when the server is busy and Figure 1b: Expected number of units in the queue when the server is on vacation for different values of λ and η, when $\gamma = 0.3$

Figure 2: Expected number of units in the queue when $\gamma = 0.5$

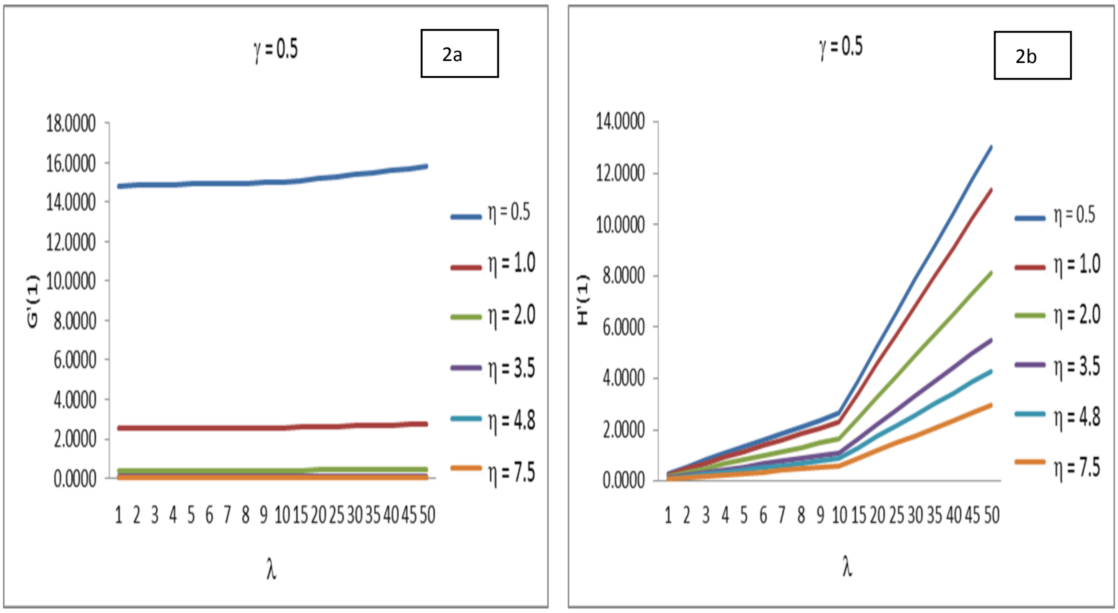

Figure 2a: Expected number of units in the queue when the server is busy and Figure 2b: Expected number of units in the queue when the server is on vacation for different values of λ and η, when $\gamma = 0.5$

Figure 3: Expected number of units in the queue when $\gamma = 0.8$

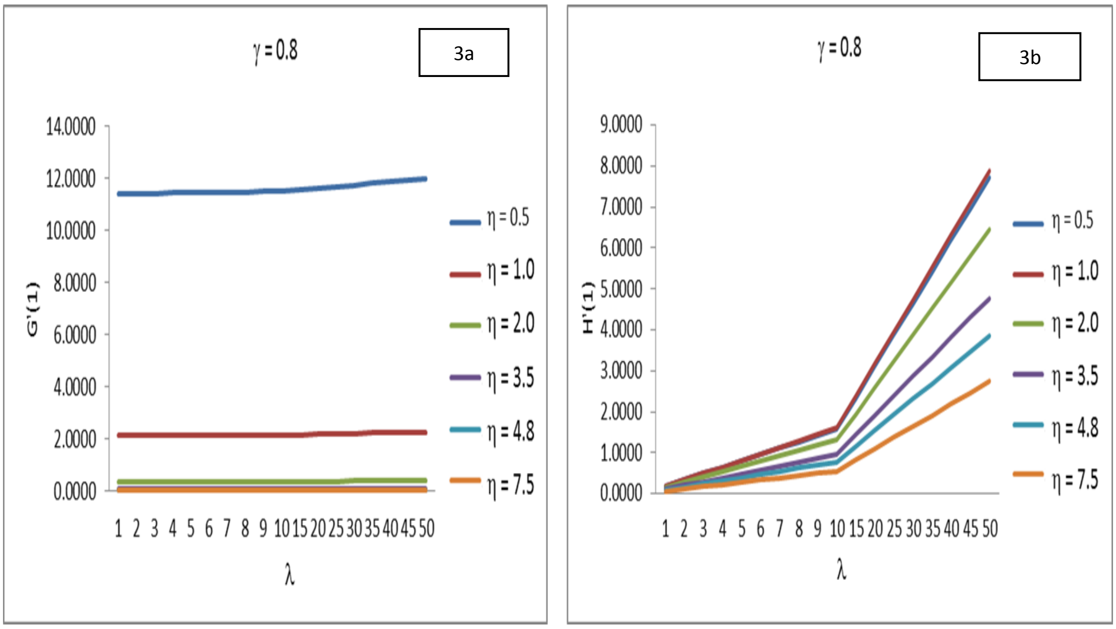

Figure 3a: Expected number of units in the queue when the server is busy and Figure 3b: Expected number of units in the queue when the server is on vacation for different values of λ and η, when $\gamma = 0.8$

Figure 4: Expected number of units in the queue when $\gamma = 1.2$

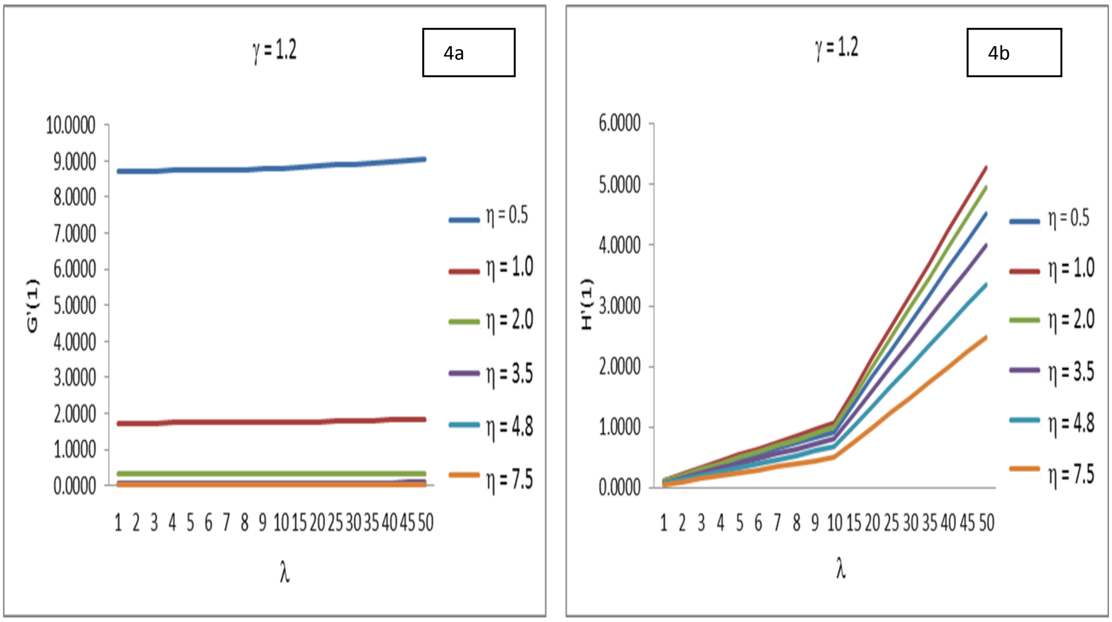

Figure 4a: Expected number of units in the queue when the server is busy and Figure 4b: Expected number of units in the queue when the server is on vacation for different values of λ and η, when $\gamma = 1.2$

Figure 5: Expected number of units in the queue when γ = 1.7

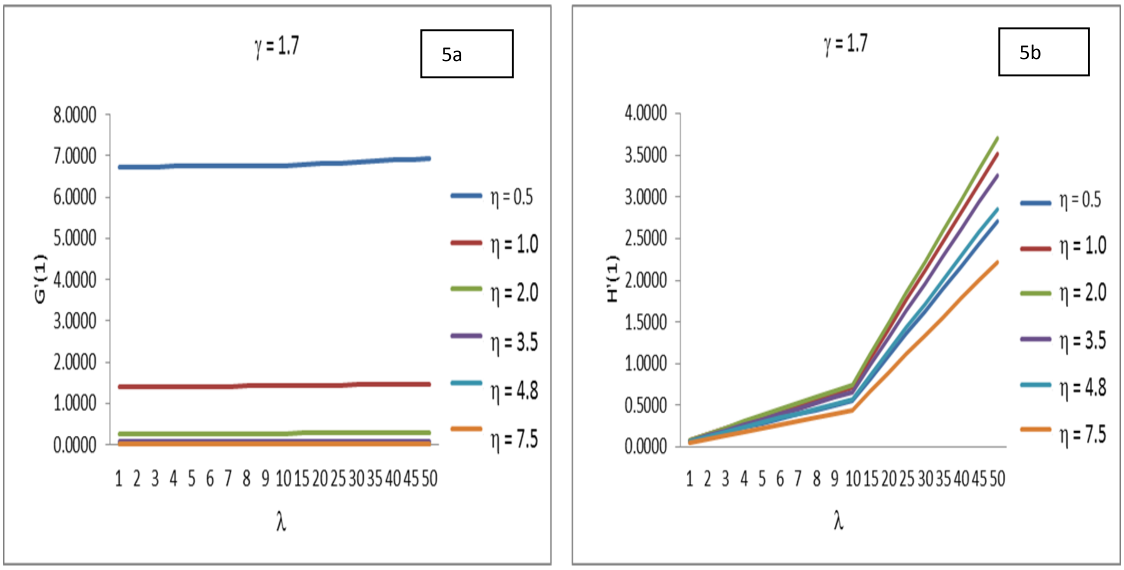

Figure 5a: Expected number of units in the queue when the server is busy and Figure 5b: Expected number of units in the queue when the server is on vacation for different values of λ and η, when γ = 1.7

7. CONCLUSION

A single server Markovian queue is analysed through the concepts like bulk arrival, bulk service, catastrophe and vacation. By applying probability generating functions, the mathematical expressions for mean queue sizes, factorial moments and variances are derived under the condition that the server is busy and that on vacation.

The numerical results for mean queue sizes are computed when the server is busy and that on vacation based on catastrophe, vacation and arrival rates. These results and their corresponding figures are presented in Tables 1-10 and Figures 1-5 respectively.

On considering fixed vacation rate, under busy service, the Figures 1a, 2a, 3a, 4a, 5a reveals that the expected number of units in the queue decreases rapidly when catastrophe rates increases with fixed mean arrival rate. Again, for fixed catastrophe rate, mean queue size increases when the mean arrival rate increases. On the other hand, for fixed catastrophe and mean arrival rates, mean queue size decreases when vacation rate increases.

Similarly in the case of server is on vacation with fixed vacation rate, the Figures 1b, 2b, 3b, 4b, 5b shows that mean number of units in the queue increases for increasing mean arrival rate with fixed catastrophe rate. It is observed that the mean queue sizes highly increase at λ = 10 and onwards due to high increase of mean arrival rates. Also, mean queue size decreases when catastrophe rate increases with fixed mean arrival rate.

Finally, it is observed that for varying vacation rate with fixed catastrophe and mean arrival rates, the mean number of units in the queue decreases. The derived results in this paper are very much applicable to network communications. It is suggested that this present work may be extended by applying different queueing techniques such as breakdown analysis, machine repair model, retrial policy, feedback rule and so on.

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