Coupled Fixed Point Theorem for Mapping Satisfying Integral Type Contraction on Cone Metric Space

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Abstract

In this paper, the existence and uniqueness of coupled fixed point theorem for mapping satisfying integral type contraction on cone metric space has been proved. The result is verified with the help of suitable example.

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1. INTRODUCTION AND MATHEMATICAL PRELIMINARIES

In 2002, Branciari [13] gave a modified version of the Banach contraction principle which is mainly integral type inequality for a single mapping and showed the following memorable fixed point theorem.

Theorem 1. *Let (X, d) be a complete metric space, $a \in (0, 1)$, and let $f : X \to X$ be a mapping such that for each $x, y \in X$,*

$$\int_0^{d(fx,fy)} \varphi(t)dt \le a \int_0^{d(x,y)} \varphi(t)dt, \tag{1}$$

where $\varphi : [0, +\infty) \to [0, +\infty)$ is a Lebesgue-integrable mapping which is summable on each compact subset of $[0, +\infty)$, nonnegative, and such that

$$\forall \epsilon > 0, \quad \int_0^{\epsilon} \varphi(t)dt > 0.$$

Then, f admits a unique fixed point $p \in X$ such that for each $x \in X$, $\lim_{n \to \infty} f^n x = p$. Theorem 1 is a generalization of the Banach-Caccioppoli [8] principle.

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In 2006, The concept of coupled fixed point was given by Bhaskar and Lakshmikantham in [2]. After that several authors proved various coupled and common coupled fixed point theorem in partial ordered metric space, G-metric space, b-metric space, fuzzy metric space, cone metric space etc. Some of the works are noted in [4], [5], [9].

In 2007, Huang and Zhang [1] introduced the concept of cone metric space, where they generalized metric space by replacing the set of real numbers with an ordering Banach space. Thus, the cone naturally induces a partial order in Banach space. Some of the works are noted in [6], [7], [10], [11].

The aim of this paper is to establish the existence and uniqueness of coupled fixed point theorem satisfying integral type contraction in cone metric space. In this paper we do not impose the normality condition of the cone. The only assumption is that cone P has nonempty interior.

Some definitions and results are as follows:

Definition 2. [1] Let E be a real Banach space and P be a subset of E. P is called a cone if and only if:

  (i) P is nonempty, closed, and $P \neq \{0\}$;

  (ii) $a, b \in \mathbb{R}, \ \ a, b \geq 0; \ \ x, y \in P \Longrightarrow ax + by \in P$;

  (iii) $x \in P$ and $-x \in P \Longrightarrow x = 0$;

Given a cone $P \subset E$, a partial ordering $\leq$ with respect to P is naturally defined by $x \leq y$ if and only if $x - y \in P$ for $x, y \in E$. We shall write $x < y$ to indicate that $x \leq y$ but $x \neq y$, while $x \ll y$ will stand for $y - x \in \text{int} P$, where $\text{int} P$ denote the interior of P.

The cone P is said to be normal if there exists a real number $K > 0$ such that for all $x, y \in E$,

$$0 \leq x \leq y \longrightarrow \|x\| \leq K\|y\|.$$

The least positive number K satisfying the above statement is called the normal constant of P.

The cone P is called regular if every increasing sequence which is bounded from above is convergent; that is, if $\{x_n\}$ is a sequence such that

$$x_1 \leq x_2 \leq \cdots \leq x_n \leq \cdots \leq y,$$

for some $y \in E$, then there is $x \in E$ such that $\|x_n - x\| \longrightarrow 0$ as $n \longrightarrow \infty$. Equivalently, the cone P is regular if and only if every decreasing sequence which is bounded from below is convergent. It is well known that a regular cone is a normal cone.

In the following we always suppose that E is a real Banach space with cone P in E with $\text{int} P \neq \emptyset$ and $\leq$ is partial ordering with respect to P.

Definition 3. [1] Let X be a nonempty set. Let the mapping $d : X \times X \longrightarrow E$ satisfies:

(i) $0 \leq d(x, y)$, for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$;

(ii) $d(x, y) = d(y, x)$, for all $x, y \in X$;

(iii) $d(x, y) \leq d(x, z) + d(z, y)$, for all $x, y, z \in X$.

Then d is called a cone metric on X and (X, d) is called a cone metric space.

Definition 4. [1] Let (X, d) be a cone metric space, $\{x_n\}$ a sequence in X and $x \in X$.

(i) If for every $c \in E$ with $0 \ll c$, there exists $n_0 \in \mathbb{N}$ such that for all $n > n_0$, $d(x_n, x) \ll c$, then $\{x_n\}$ is said to be convergent and $\{x_n\}$ converges to x, and x is the limit of $\{x_n\}$. This limit is denoted by $\lim_n x_n = x$ or $x_n \longrightarrow x$ as $n \longrightarrow \infty$.

(ii) If for every $c \in E$ with $0 \ll c$, there exists $n_0 \in N$ such that for all $n, m > n_0$, $d(x_n, x_m) \ll c$, then $\{x_n\}$ is called a Cauchy sequence in X.

(iii) If every Cauchy sequence in X is convergent in X, then X is called a complete cone metric space.

Lemma 5. *[6] If E be a real Banach space with cone P in E, then for $a, b, c \in E$,*

(i) *If $a \leq b$ and $b \ll c$, then $a \ll c$.*

(ii) *If $a \ll b$ and $b \ll c$, then $a \ll c$.*

Definition 6. [2] An element $(x, y) \in X \times X$ is called a coupled fixed point of the mapping $F : X \times X \longrightarrow X$ if $x = F(x, y)$ and $y = F(y, x)$.

2. MAIN RESULTS

Theorem 7. *Let (X, d) be a complete cone metric space with a cone P having nonempty interior. Let $F : X \times X \longrightarrow X$ satisfying*

$$\int_0^{d(F(x,y),F(u,v))} \phi(t)dt \leq \alpha \int_0^{d(x,u)} \phi(t)dt + \beta \int_0^{d(y,v)} \phi(t)dt + \gamma \int_0^{d(u,F(x,y))} \phi(t)dt$$
$$+ \delta \int_0^{d(u,F(u,v))} \phi(t)dt, \tag{2}$$

for all $x, y, u, v \in X$, and $\alpha, \beta, \gamma, \delta \in (0, 1)$ such that $\alpha + \beta + \gamma + \delta < 1$. and $\phi(t) : P \to P$ is a Lebesgue-integral mapping which is summable, nonnegative and

such that for all $0 \ll \epsilon$, $0 \ll \int_0^\epsilon \phi(t)dt$.
Then $F(x,y)$ *has a unique coupled fixed point in* $X \times X$.

Proof. Let x_0 and y_0 be two arbitrary points in X.
Let

$$x_{2k+1} = F(x_{2k}, y_{2k}), \qquad\qquad y_{2k+1} = F(y_{2k}, x_{2k}).$$

$$x_{2k+2} = F(x_{2k+1}, y_{2k+1}), \qquad\qquad y_{2k+2} = F(y_{2k+1}, x_{2k+1}).$$

for k = 0, 1, 2, 3, ... Then

$$
\begin{aligned}
\int_0^{d(x_{2k+1}, x_{2k+2})} \phi(t)dt &= \int_0^{d(F(x_{2k}, y_{2k}), F(x_{2k+1}, y_{2k+1}))} \phi(t)dt \\
&\le \alpha \int_0^{d(x_{2k}, x_{2k+1})} \phi(t)dt + \beta \int_0^{d(y_{2k}, y_{2k+1})} \phi(t)dt \\
&\quad + \gamma \int_0^{d(x_{2k+1}, F(x_{2k}, y_{2k}))} \phi(t)dt + \delta \int_0^{d(x_{2k+1}, F(x_{2k+1}, y_{2k+1}))} \phi(t)dt \\
&\le \alpha \int_0^{d(x_{2k}, x_{2k+1})} \phi(t)dt + \beta \int_0^{d(y_{2k}, y_{2k+1})} \phi(t)dt \\
&\quad + \gamma \int_0^{d(x_{2k+1}, x_{2k+1})} \phi(t)dt + \delta \int_0^{d(x_{2k+1}, x_{2k+2})} \phi(t)dt.
\end{aligned}
$$

Hence

$$\int_0^{d(x_{2k+1}, x_{2k+2})} \phi(t)dt \le \frac{\alpha}{(1-\delta)} \int_0^{d(x_{2k}, x_{2k+1})} \phi(t)dt + \frac{\beta}{(1-\delta)} \int_0^{d(y_{2k}, y_{2k+1})} \phi(t)dt.$$

$$(3)$$

Similarly, we have

$$
\begin{aligned}
\int_0^{d(y_{2k+1}, y_{2k+2})} \phi(t)dt &= \int_0^{d(F(y_{2k}, x_{2k}), F(y_{2k+1}, x_{2k+1}))} \phi(t)dt \\
&\le \alpha \int_0^{d(y_{2k}, y_{2k+1})} \phi(t)dt + \beta \int_0^{d(x_{2k}, x_{2k+1})} \phi(t)dt \\
&\quad + \gamma \int_0^{d(y_{2k+1}, F(y_{2k}, x_{2k}))} \phi(t)dt + \delta \int_0^{d(y_{2k+1}, F(y_{2k+1}, x_{2k+1}))} \phi(t)dt \\
&\le \alpha \int_0^{d(y_{2k}, y_{2k+1})} \phi(t)dt + \beta \int_0^{d(x_{2k}, x_{2k+1})} \phi(t)dt \\
&\quad + \gamma \int_0^{d(y_{2k+1}, y_{2k+1})} \phi(t)dt + \delta \int_0^{d(y_{2k+1}, y_{2k+2})} \phi(t)dt.
\end{aligned}
$$

Hence

$$\int_0^{d(y_{2k+1},y_{2k+2})} \phi(t)dt \le \frac{\alpha}{(1-\delta)} \int_0^{d(y_{2k},y_{2k+1})} \phi(t)dt + \frac{\beta}{(1-\delta)} \int_0^{d(x_{2k},x_{2k+1})} \phi(t)dt.$$

(4)

Adding (3) and (4) we get

$$\int_0^{d(x_{2k+1},x_{2k+2})} \phi(t)dt + \int_0^{d(y_{2k+1},y_{2k+2})} \phi(t)dt \le \frac{\alpha+\beta}{(1-\delta)} \int_0^{d(x_{2k},x_{2k+1})} \phi(t)dt$$

$$+ \frac{\alpha+\beta}{(1-\delta)} \int_0^{d(y_{2k},y_{2k+1})} \phi(t)dt$$

$$\le \frac{\alpha+\beta}{(1-\delta)} \Big[\int_0^{d(x_{2k},x_{2k+1})} \phi(t)dt + \int_0^{d(y_{2k},y_{2k+1})} \phi(t)dt \Big]$$

$$= r \Big[\int_0^{d(x_{2k},x_{2k+1})} \phi(t)dt + \int_0^{d(y_{2k},y_{2k+1})} \phi(t)dt \Big],$$

(5)

where $0 < r = \frac{\alpha+\beta}{1-\delta} < 1$.
Hence, from (5) we have

$$\int_0^{d(x_{2k+2},x_{2k+3})} \phi(t)dt + \int_0^{d(y_{2k+2},y_{2k+3})} \phi(t)dt$$

$$\le r \Big[\int_0^{d(x_{2k+1},x_{2k+2})} \phi(t)dt + \int_0^{d(y_{2k+1},y_{2k+2})} \phi(t)dt \Big].$$

Therefore

$$\int_0^{d(x_n,x_{n+1})} \phi(t)dt + \int_0^{d(y_n,y_{n+1})} \phi(t)dt \le r \Big[\int_0^{d(x_{n-1},x_n)} \phi(t)dt + \int_0^{d(y_{n-1},y_n)} \phi(t)dt \Big]$$

$$\le r^2 \Big[\int_0^{d(x_{n-2},x_{n-1})} \phi(t)dt + \int_0^{d(y_{n-2},y_{n-1})} \phi(t)dt \Big]$$

$$\cdots\cdots\cdots$$

$$\le r^n \Big[\int_0^{d(x_0,x_1)} \phi(t)dt + \int_0^{d(y_0,y_1)} \phi(t)dt \Big].$$

Now if we take

$$\int_0^{d(x_n,x_{n+1})} \phi(t)dt + \int_0^{d(y_n,y_{n+1})} \phi(t)dt = \lambda_n,$$

then,

$$\lambda_n \le r\lambda_{n-1} \le \cdots \le r^n \lambda_0.$$

Let $m > n$. Then using triangular inequality,

$$d(x_m, x_n) \leq \sum_{i=n}^{m-1} d(x_i, x_{i+1}),$$

$$\int_0^{d(x_n, x_m)} \phi(t)dt + \int_0^{d(y_n, y_m)} \phi(t)dt \leq \lambda_{m-1} + \lambda_{m-2} + \cdots + \lambda_n$$
$$\leq (r^{m-1} + r^{m-2} + \cdots + r^n)\lambda_0$$
$$= r^n(1 + r + \cdots + r^{m-n-1})\lambda_0$$
$$\cdots \cdots$$
$$\leq \frac{r^n}{1-r}\lambda_0. \tag{6}$$

Consequently,

$$\int_0^{d(x_n, x_m)} \phi(t)dt \leq \frac{r^n}{1-r}\lambda_0$$
$$\int_0^{d(y_n, y_m)} \phi(t)dt \leq \frac{r^n}{1-r}\lambda_0. \tag{7}$$

Let $0 \ll c$ be given. Choose a natural number N such that $\frac{r^n}{1-r}\lambda_0 \ll c$ for $n > N$. Since for all $0 \ll \epsilon$, $0 \ll \int_0^\epsilon \phi(t)dt$ so $d(x_n, x_m) \ll c$ and $d(y_n, y_m) \ll c$ for $m > n$. Therefore both the sequences $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences.

From the completeness of X, there exist $x, y \in X$ such that

$$\lim_{n \to \infty} x_n = x \quad \text{and} \quad \lim_{n \to \infty} y_n = y.$$

Now we prove that $F(x, y) = x$ and $F(y, x) = y$. Substituting $x_{2k+1} = F(x, y)$ and $y_{2k+1} = F(y, x)$ in (5), we obtain

$$\int_0^{d(F(x,y), x_{2k+2})} \phi(t)dt + \int_0^{d(F(y,x), y_{2k+2})} \phi(t)dt$$
$$\leq r\left[\int_0^{d(x_{2k}, F(x,y))} \phi(t)dt + \int_0^{d(y_{2k}, F(y,x))} \phi(t)dt\right].$$

Letting $k \longrightarrow \infty$ we obtain

$$\int_0^{d(F(x,y), x)} \phi(t)dt + \int_0^{d(F(y,x), y)} \phi(t)dt \leq r\left[\int_0^{d(x, F(x,y))} \phi(t)dt + \int_0^{d(y, F(y,x))} \phi(t)dt\right],$$

$$\implies (1-r)\left[\int_0^{d(x, F(x,y))} \phi(t)dt + \int_0^{d(y, F(y,x))} \phi(t)dt\right] \leq 0.$$

Since $0 < r < 1$ so $(1 - r) > 0$ therefore

$$\int_0^{d(x,F(x,y))} \phi(t)dt + \int_0^{d(y,F(y,x))} \phi(t)dt \le 0.$$

Which is a contradiction.

Therefore,

$$\int_0^{d(x,F(x,y))} \phi(t)dt + \int_0^{d(y,F(y,x))} \phi(t)dt = 0.$$

Since for all $0 \ll \epsilon$, $0 \ll \int_0^\epsilon \phi(t)dt$ so we have

$\implies \qquad d(F(x,y),x) = 0$ and $d(F(y,x),y) = 0$,

so $\qquad F(x,y) = x$ and $F(y,x) = y$.

In order to prove the uniqueness let (x',y') be another point of $X \times X$ such that $x' = F(x',y')$ and $y' = F(y',x')$. Then

$$\int_0^{d(x,x')} \phi(t)dt = \int_0^{d(F(x,y),F(x',y'))} \phi(t)dt$$

$$\le \alpha \int_0^{d(x,x')} \phi(t)dt + \beta \int_0^{d(y,y')} \phi(t)dt$$

$$+ \gamma \int_0^{d(x',F(x,y))} \phi(t)dt + \delta \int_0^{d(x',F(x',y'))} \phi(t)dt$$

$$\le \alpha \int_0^{d(x,x')} \phi(t)dt + \beta \int_0^{d(y,y')} \phi(t)dt$$

$$+ \gamma \int_0^{d(x',x)} \phi(t)dt + \delta \int_0^{d(x',x')} \phi(t)dt.$$

$$\implies (1 - \alpha - \gamma) \int_0^{d(x,x')} \phi(t)dt \le \beta \int_0^{d(y,y')} \phi(t)dt.$$

$$\implies \int_0^{d(x,x')} \phi(t)dt \le \frac{\beta}{(1 - \alpha - \gamma)} \int_0^{d(y,y')} \phi(t)dt. \qquad (8)$$

Similarly, we can prove that

$$\int_0^{d(y,y')} \phi(t)dt \le \frac{\beta}{(1 - \alpha - \gamma)} \int_0^{d(x,x')} \phi(t)dt. \qquad (9)$$

Adding (8) and (9) we get

$$\int_0^{d(y,y')} \phi(t)dt + \int_0^{d(x,x')} \phi(t)dt \le \frac{\beta}{(1 - \alpha - \gamma)} \left[\int_0^{d(x,x')} \phi(t)dt + \int_0^{d(y,y')} \phi(t)dt \right].$$

$$\implies \left(1 - \frac{\beta}{1 - \alpha - \gamma}\right) \left[\int_0^{d(y,y')} \phi(t)dt + \int_0^{d(x,x')} \phi(t)dt \right] \le 0.$$

Since $0 < \left(1 - \frac{\beta}{1-\alpha-\gamma}\right) < 1$ so

$$\int_0^{d(y,y')} \phi(t)dt + \int_0^{d(x,x')} \phi(t)dt \leq 0,$$

$$\implies \int_0^{d(y,y')} \phi(t)dt + \int_0^{d(x,x')} \phi(t)dt = 0,$$

i.e $d(x,x') = 0$ and $d(y,y') = 0$,

i.e $x = x'$ and $y = y'$.

Now the proof is completed.   □

Corollary 8. *In Theorem 7 if we take $\gamma = 0$ and $\delta = 0$. Then we have,*
Let (X,d) be a complete cone metric space with a cone P having nonempty interior.
Let $F : X \times X \longrightarrow X$ satisfying

$$\int_0^{d(F(x,y),F(u,v))} \phi(t)dt \leq \alpha \int_0^{d(x,u)} \phi(t)dt + \beta \int_0^{d(y,v)} \phi(t)dt,$$

for all $x, y, u, v \in X$, and $\alpha, \beta \in (0,1)$ such that $\alpha + \beta < 1$ and $\phi(t) : P \to P$ is a Lebesgue-integral mapping which is summable, nonnegative and such that for all $0 \ll \epsilon, 0 \ll \int_0^\epsilon \phi(t)dt$.
Then $F(x,y)$ has a unique coupled fixed point in $X \times X$.

Corollary 9. *In Theorem 7 if we take $\delta = 0$. Then we have,*
Let (X,d) be a complete cone metric space with a cone P having nonempty interior.
Let $F : X \times X \longrightarrow X$ satisfying

$$\int_0^{d(F(x,y),F(u,v))} \phi(t)dt \leq \alpha \int_0^{d(x,u)} \phi(t)dt + \beta \int_0^{d(y,v)} \phi(t)dt + \gamma \int_0^{d(u,F(x,y))} \phi(t)dt,$$

for all $x, y, u, v \in X$, and $\alpha, \beta, \gamma \in (0,1)$ such that $\alpha + \beta + \gamma < 1$ and $\phi(t) : P \to P$ is a Lebesgue-integral mapping which is summable, nonnegative and such that for all $0 \ll \epsilon, 0 \ll \int_0^\epsilon \phi(t)dt$.
Then $F(x,y)$ has a unique coupled fixed point in $X \times X$.

Corollary 10. *In Theorem 7 if we take $\gamma = 0$. Then we have,*
Let (X,d) be a complete cone metric space with a cone P having nonempty interior.
Let $F : X \times X \longrightarrow X$ satisfying

$$\int_0^{d(F(x,y),F(u,v))} \phi(t)dt \leq \alpha \int_0^{d(x,u)} \phi(t)dt + \beta \int_0^{d(y,v)} \phi(t)dt + \delta \int_0^{d(u,F(u,v))} \phi(t)dt,$$

for all $x, y, u, v \in X$, and $\alpha, \beta, \delta \in (0,1)$ such that $\alpha + \beta + \delta < 1$ and $\phi(t) : P \to P$ is a Lebesgue-integral mapping which is summable, nonnegative and such that for all $0 \ll \epsilon, 0 \ll \int_0^\epsilon \phi(t)dt$.
Then $F(x,y)$ has a unique coupled fixed point in $X \times X$.

3. EXAMPLE

Let $E = \mathbb{R}$ and $P = \{x \in E : x \geq 0\}$. Let $X = [0, 2]$ and define a mapping $d : X \times X \to X$ by $d(x, y) = |x - y|$. Then (X, d) is a complete cone metric space. Define $F : X \times X \to X$ by $F(x, y) = \frac{x+y}{2}$ for all $x, y \in X$. Define $\phi(t) : P \to P$ by $\phi(t) = t$ for all $t \in P$.

Then the condition of the theorem 7 is satisfied.

Here it is seen that $(\frac{1}{2}, \frac{1}{2})$ is the unique coupled fixed point of $F(x, y)$.

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