Advances in Dynamical Systems and Applications (ADSA).
ISSN 0973-5321, Volume 14, Number 2, (2019) pp. 281-290

http://www.ripublication.com
https://dx.doi.org/10.37622/ADSA/14.2.2019.281-290

MTSF Analysis of 1-out-of-2 Cold Standby Systems with Continuous Operation Subject to Weather Conditions

Dr. Savita Deswal

Assistant Professor Mathematics

Government P. G. College for Women Rohtak, Haryana, India

Email - *sdeswal2981@gmail.com*

Abstract

In this paper, I propose a system of two dissimilar units: one unit is called as main unit, and the other is called as duplicate unit. At any time, one of the two units is operating while the other is on cold standby. The single repairman is not available to perform the job in abnormal weather condition while the system remains operative in all weather conditions. The failure times and the time to change the weather follow exponentials distribution whereas repair rates are arbitrarily distributed. Under these assumptions, we investigate the reliability measures of the system using the regenerative point technique. The paper derives a number of reliability indices: system reliability, mean time to system failure, Transition Probabilities and Mean Sojourn Times.

Index Terms — Reliability Measures, Standby Unit, Weather Conditions, Regenerative Point Technique, Mean Time to System Failure, Sojourn time

I. INTRODUCTION

In our ever-evolving world, the reliability of systems and processes has become a paramount concern. From the intricate machinery powering modern industries to the critical infrastructure supporting our daily lives, the ability of these systems to perform consistently and dependably is essential. The weather conditions contribute significantly to the performance of the operating systems. The physical stresses created by adverse weather conditions deteriorate performance and efficiency of the systems. Besides, scientists and engineers have tried a lot to develop systems which can work in varying environmental conditions and the performance of such systems has been improved up to a considerable level by adopting better repair policies and redundancy techniques. Therefore, a stochastic Models of a system of non-identical units have been analyzed under two weather conditions-normal and abnormal. Sometimes we may have emergency situations in which operation of the system becomes necessary irrespective of weather conditions. Goel et. al. (1985) examined the two-unit cold standby system. Gupta, R., and Goel, R. (1991) analyzed the profit under abnormal weather conditions. Malik et. al. (2012) performed the stochastic analysis of a repairable system of non-identical units with priority for operation and repair subject to weather conditions. Kumar and Goel (2016) evaluated availability and profit for general distribution.

This research paper embarks on a journey into the realm of reliability analysis, aiming to strengthen the existing literature on reliability and also to know the variations in reliability and economic measures of a system of non-identical units operating in different weather conditions. I will delve into the core concepts that govern reliability, including failure modes and effects, statistical modeling, and data-driven approaches. To meet out this objective a stochastic Model is developed under different set of assumptions on operation and repair policies. Initially, one original unit (called main unit) is operative and the other substandard unit (called duplicate unit) is kept at spare in cold standby. Each unit has constant failure unit from normal mode. It is assumed that both units are capable of performing the same set of functions and activities but with different proficiencies. The system operates in two weather conditions - normal and abnormal. However, repair of the system is allowed only in normal weather by a server visits the system immediately as and when needed. The distribution of failure times of units and change of weather conditions are taken as negative exponential while that of repair times of the units follow arbitrary distributions. The units work as new after repair. All random variables are statistically independent. The semi-Markov and regenerative point technique are adopted to drive the expressions for the reliability measures such as mean time to system failure (MTSF). The results are analyzed through graphs for particular values of various parameters and costs.

II. SYSTEM DESCRIPTION

The system is described as below –

System	Repairable System with two non-identical units as main unit and duplicate unit. Each unit is capable of performing same set of activities but with different efficiency
Operation	Initially main unit is operative and duplicate unit is at standby
Weather Conditions	Normal and Abnormal weather
Redundancy	Cold Standby
Mode	Unit is either operative in normal mode or failed
Server	Single server with immediate arrival in normal weather
Switches & Repair	Perfect and Instantaneous

III. SYSTEM TRANSITION STATES

The different system transition states are defined as below

S_0 = (MO, DCs), Main unit operative and duplicate at cold standby in normal weather

S_1 = (MFur, DO), Main unit failed under repair and duplicate operative in normal weather

S_2 = ($\overline{MO}$, $\overline{DCs}$), Main unit operative and duplicate at cold standby in abnormal weather

S_3= ($\overline{MFwr}$, $\overline{DO}$), Main unit failed waiting for repair and duplicate operative in abnormal weather

S_4= (MFUR, DFwr), Main unit failed continuously under repair and duplicate failed waiting for repair in normal weather

S_5= (MCs, DO), Main unit at cold standby and duplicate operative in normal weather

S_6= (MO, DFur), Main unit operative and duplicate failed under repair in normal weather

S_7= ($\overline{MFwr}$, $\overline{DFWR}$), Main unit failed waiting for repair and duplicate failed waiting for repair continuously in abnormal weather

S_8= (MFur, DFWR), Main unit failed under repair and duplicate failed waiting for repair in normal weather

S_9= ($\overline{MCs}$, $\overline{DWO}$), Main unit at cold standby and duplicate waiting for operation in abnormal weather

S_{10}= ($\overline{MWO}$, $\overline{DFwr}$), Main unit waiting for operation and duplicate waiting for repair in abnormal weather

S_{11}= (MFwr, DFUR), Main unit failed waiting for repair and duplicate failed under repair continuously in normal weather

S_{12}= ($\overline{MFWR}$, $\overline{DFwr}$), Main unit failed waiting for repair continuously and duplicate failed waiting for repair in abnormal weather

The up states in this system are S_0, S_1, S_2, S_3, S_5, S_7, S_{10} and S_{12} and the down states are S_4, S_6, S_8, S_9 and S_{11}. In addition, the states S_0, S_1, S_2, S_3, S_5, S_7, S_{10} and S_{12} are regenerative while the states S_4, S_6, S_8, S_9 and S_{11} are non-regenerative.

IV. TRANSITION PROBABLITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements

$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$, we have

$p_{01}=\frac{\lambda}{\beta+\lambda}$, $p_{02}=\frac{\beta}{\beta+\lambda}$, $p_{13}=\frac{\beta}{\beta+\lambda_1}$ (1-g*(β+λ1)), $p_{14}=\frac{\lambda_1}{\beta+\lambda_1}$(1-g*(β+λ1)), p_{15}= g*(β+λ1), $p_{20}=\frac{\beta_1}{\beta_1+\lambda}$, $p_{23}=\frac{\lambda}{\beta_1+\lambda}$,

$p_{31}=\frac{\beta_1}{(\beta_1+\lambda)}$, $p_{36}=\frac{\lambda_1}{(\beta_1+\lambda_1)}$, p_{47}=g*(β), p_{48}=1-g*(β), $p_{57}=\frac{\lambda_1}{(\lambda_1+\beta)}$, $p_{5,10}=\frac{\beta}{(\lambda_1+\beta)}$, p_{69}=1, p_{70}=g1*(β+λ),

$p_{7,11}=\frac{\lambda}{\beta+\lambda}$ (1-g1*(β+λ)), $p_{7,12}=\frac{\beta}{\beta+\lambda}$ (1-g1*(β+λ)), p_{89}=1, p_{97}=g*(β), p_{98}=1-g*(β), $p_{10,5}=\frac{\beta_1}{(\beta_1+\lambda_1)}$,

$p_{10,12}=\frac{\lambda_1}{(\beta_1+\lambda_1)}$, $p_{11,1}$=g1*(β), $p_{11,6}$=1-g*(β), $p_{12,7}=\frac{\beta_1}{(\beta_1+\lambda)}$, $p_{12,8}=\frac{\lambda}{(\beta_1+\lambda)}$

The mean sojourn times (μ_i) in the state S_i are

$\mu_0=m_{01}+m_{02}=\frac{1}{\lambda+\beta}$, $\mu_1= m_{13}+m_{14}+m_{15}=\frac{1}{(\lambda_1+\beta)}$(1-g*(β+λ1)) , $\mu_2 = m_{20}+m_{23}=\frac{1}{\lambda+\beta_1}$, $\mu_3= m_{31}+m_{36}=\frac{1}{\lambda_1+\beta_1}$,

$\mu_4= m_{47}+m_{48}=\frac{1}{\beta}$ (1-g*(β)), $\mu_5= m_{57}+ m_{5,10}=\frac{1}{\lambda_1+\beta}$, $\mu_6= m_{69}=\frac{1}{\beta_1}$, $\mu_7=m_{70}+m_{7,11}+m_{7,12}=\frac{1}{(\lambda+\beta)}$ (1-g1*(β+λ)),

$\mu_8=m_{89}=\frac{1}{\beta_1}$,$\mu_9=m_{97}+m_{98}=\frac{1}{\beta}$ (1-g*(β)), $\mu_{10}=m_{10,5}+m_{10,12}=\frac{1}{\lambda_1+\beta_1}$, $\mu_{11}=m_{11,1}+m_{11,6}=\frac{1}{\beta}$ (1-g1*(β)),

$\mu_{12}=m_{12,7}+m_{12,8}=\frac{1}{\lambda+\beta_1}$

V. RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let $\emptyset_i(t)$ be the cdf of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\emptyset_i(t)$:

$$\phi_i(t) = \sum_j Q_{i,j}(t) Ⓢ \phi_j(t) + \sum_j Q_{i,j}(t)$$

Where S_j is an un-failed regenerative state to which the given regenerative state S_i can transit and S_k is a failed state to which the state S_i can transit directly.

Taking LST of the above relations and solving $\phi_0^{**}(s) = \frac{N_1(s)}{D_1(s)}$

$$N_1(s)=\begin{vmatrix} 0 & -Q_{01}^{**}(s) & -Q_{02}^{**}(s) & 0 & 0 & 0 & 0 & 0 \\ Q_{14}^{**}(s) & 1 & 0 & -Q_{13}^{**}(s) & -Q_{15}^{**}(s) & 0 & 0 & 0 \\ 0 & 0 & 1 & -Q_{23}^{**}(s) & 0 & 0 & 0 & 0 \\ Q_{36}^{**}(s) & -Q_{31}^{**}(s) & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -Q_{57}^{**}(s) & -Q_{5,10}^{**}(s) & 0 \\ Q_{7,11}^{**}(s) & 0 & 0 & 0 & 0 & 1 & 0 & -Q_{7,12}^{**}(s) \\ 0 & 0 & 0 & 0 & -Q_{10,5}^{**}(s) & 0 & 1 & -Q_{10,12}^{**}(s) \\ Q_{12,8}^{**}(s) & 0 & 0 & 0 & 0 & -Q_{12,7}^{**}(s) & 0 & 1 \end{vmatrix}$$

$$D_1(s)=\begin{vmatrix} 1 & -Q_{01}^{**}(s) & -Q_{02}^{**}(s) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -Q_{13}^{**}(s) & -Q_{15}^{**}(s) & 0 & 0 & 0 \\ -Q_{20}^{**}(s) & 0 & 1 & -Q_{23}^{**}(s) & 0 & 0 & 0 & 0 \\ 0 & -Q_{31}^{**}(s) & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -Q_{57}^{**}(s) & -Q_{5,10}^{**}(s) & 0 \\ -Q_{70}^{**}(s) & 0 & 0 & 0 & 0 & 1 & 0 & -Q_{7,12}^{**}(s) \\ 0 & 0 & 0 & 0 & -Q_{10,5}^{**}(s) & 0 & 1 & -Q_{10,12}^{**}(s) \\ 0 & 0 & 0 & 0 & 0 & -Q_{12,7}^{**}(s) & 0 & 1 \end{vmatrix}$$

We have $R^*(s) = \frac{1-\phi_0^{**}(s))}{s}$

The reliability of the system Model can be obtained by taking inverse Laplace transform of above equation.

The mean time to system failure (MTSF) is given by

$$MTSF = \lim_{s\to 0} R^*(s) \quad = \frac{N_{11}}{D_{11}}$$

$N_{11}= (1-p_{7,12}p_{12,7}) (1-p_{5,10}p_{10,5}) ((p_{02}\mu_2+\mu_0) (1-p_{13}p_{31}) + \mu_1(p_{01}+p_{02}p_{23}p_{31}) + \mu_3(p_{02}p_{23}+p_{01}p_{13}))$

$+ p_{15}(p_{01}+p_{02}p_{23}p_{31}) ((\mu_5+\mu_{10}p_{5,10}) (1-p_{7,12}p_{12,7}) + \mu_7(p_{5,7}+p_{5,10}p_{10,12}p_{12,7}) + \mu_{12} (p_{57}p_{7,12}+p_{5,10}p_{10.12}))$

$D_{11}= (1-p_{7,12}p_{12,7}) (1-p_{5,10}p_{10,5}) (1-p_{02}p_{20}) (1-p_{13}p_{31}) - p_{70}p_{15}(p_{01}+p_{02}p_{23}p_{31}) (p_{57}+p_{5,10}p_{10,12}p_{12,7})$

VI. PARTICULAR CASES

Suppose $g(t) = \alpha e^{-\alpha t}$,$g_1(t) = \alpha_1 e^{-\alpha_1 t}$

By using the non-zero elements p_{ij}, we can obtain the following results:

$p_{01}=\frac{\lambda}{(\beta+\lambda)}$, $p_{02}=\frac{\beta}{(\beta+\lambda)}$, $p_{13}=\frac{\beta}{(\beta+\alpha+\lambda_1)}$, $p_{14}=\frac{\lambda_1}{(\beta+\alpha+\lambda_1)}$, $p_{15}=\frac{\alpha}{(\beta+\alpha+\lambda_1)}$, $p_{20}=\frac{\beta_1}{(\beta_1+\lambda)}$, $p_{23}=\frac{\lambda}{(\beta_1+\lambda)}$, $p_{31}=\frac{\beta_1}{(\beta_1+\lambda_1)}$,

$p_{36}=\frac{\lambda_1}{(\beta_1+\lambda_1)}$, $p_{47}=\frac{\alpha}{(\alpha+\beta)}$, $p_{48}=\frac{\alpha}{(\alpha+\beta)}$, $p_{57}=\frac{\lambda_1}{(\lambda_1+\beta)}$, $p_{5,10}=\frac{\beta}{(\lambda_1+\beta)}$,$p_{69}=1$, $p_{70}=\frac{\alpha_1}{(\beta+\alpha_1+\lambda)}$, $p_{7,11}=\frac{\lambda}{(\beta+\alpha_1+\lambda)}$,

$p_{7,12}=\frac{\beta}{(\beta+\alpha_1+\lambda)}$, $p_{89}=1$, $p_{97}=\frac{\alpha}{(\alpha+\beta)}$,$p_{98}=\frac{\beta}{(\alpha+\beta)}$, $p_{10,5}=\frac{\beta_1}{(\beta_1+\lambda_1)}$, $p_{10,12}=\frac{\lambda_1}{(\beta_1+\lambda_1)}$, $p_{11,1}=\frac{\alpha_1}{(\alpha_1+\beta)}$, $p_{11,6}=\frac{\beta}{(\alpha_1+\beta)}$,

$p_{12,7}=\frac{\beta_1}{(\beta_1+\lambda)}$, $p_{12,8}=\frac{\lambda}{(\beta_1+\lambda)}$

$\mu_0=\frac{1}{(\beta+\lambda)}$, $\mu_1=\frac{1}{(\alpha+\lambda_1+\beta)}$, $\mu_2=\frac{1}{(\beta_1+\lambda)}$, $\mu_3=\frac{1}{\lambda_1+\beta_1}$, $\mu_4=\frac{1}{(\alpha+\beta)}$, $\mu_5=\frac{1}{(\lambda_1+\beta)}$, $\mu_6=\frac{1}{(\beta_1)}$, $\mu_7=\frac{1}{(\lambda+\alpha_1+\beta)}$, $\mu_8=\frac{1}{(\beta_1)}$,

$\mu_9=\frac{1}{(\beta+\alpha)}$, $\mu_{10}=\frac{1}{\lambda_1+\beta_1}$, $\mu_{11}=\frac{1}{(\alpha_1+\beta)}$, $\mu_{12}=\frac{1}{(\beta_1+\lambda)}$

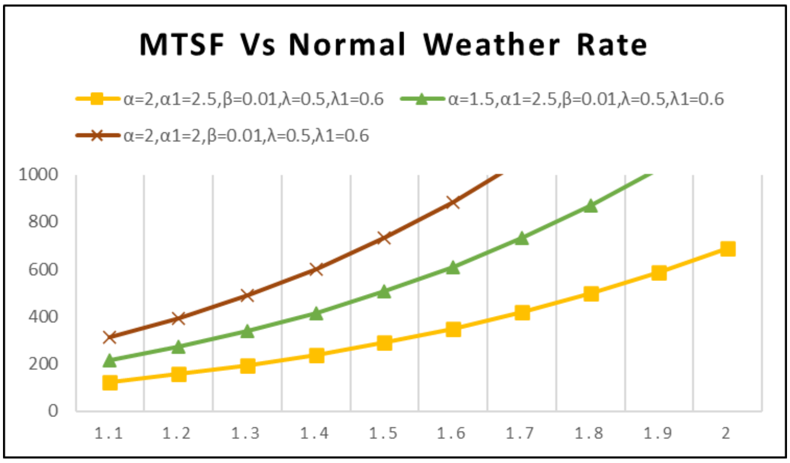
MTSF $=\frac{N_{11}}{D_{11}}$

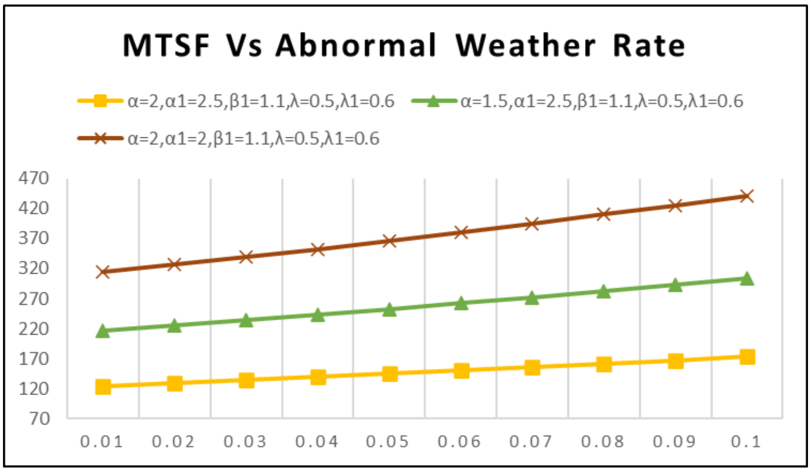
$N_{11} = ((\alpha_1+\lambda)(\beta_1+\lambda)+\beta\lambda)\,\lambda_1(\beta+\beta_1+\lambda_1)\,((\beta+\beta_1+\lambda)(E+\beta\lambda(1+C))+\alpha\beta\lambda((\beta+\beta_1+\lambda_1)A+\lambda_1B+\beta\lambda_1D)$

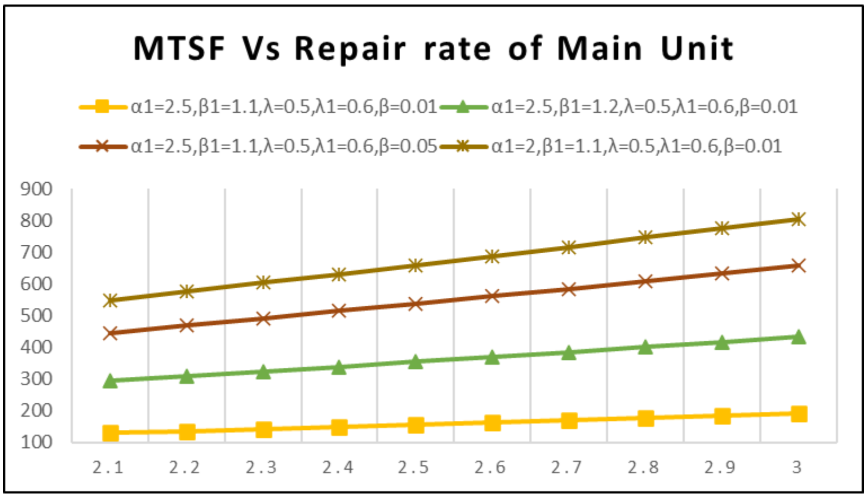
$D_{11} = \lambda\lambda_1^2((\alpha_1+\lambda)(\beta_1+\lambda)+\beta\lambda)(\beta+\beta_1+\lambda_1)(B+\beta C+\alpha\lambda\lambda_1B(\beta D+B))$

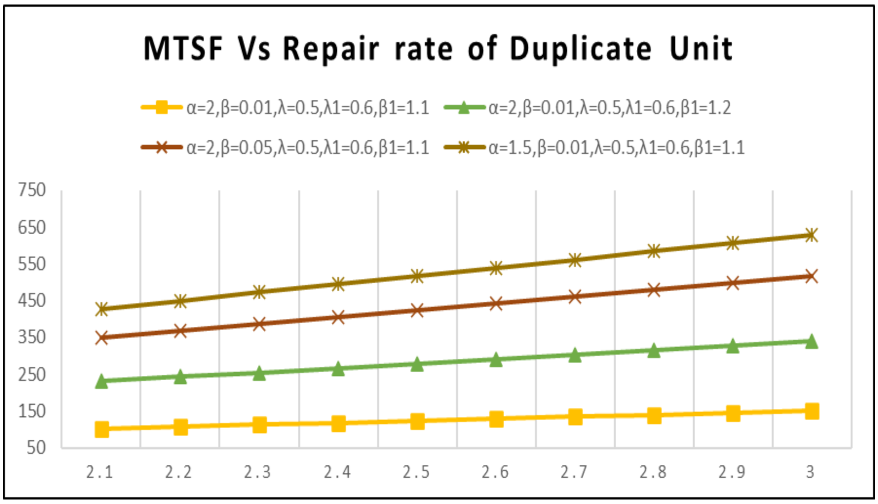
$B = (\beta_1+\lambda)(\beta_1+\lambda_1)+\beta\beta_1$, $C=\alpha+\beta+\lambda+\beta_1+\lambda_1$, $D=\alpha_1+\beta_1+\lambda_1+\beta+\lambda$, $E=(\alpha+\lambda_1)(\beta_1+\lambda_1)+\beta\lambda_1$

VII. GRAPHICAL ILLUSTRATION









VIII. CONCLUSION

The graphs for MTSF of the system are drawn for fixed values of the various parameters and costs with respect to normal weather rate, abnormal weather rate, repair rates of the either unit as shown above. These figures indicate that there is substantial positive change in these measures with increase of normal and abnormal weather rate (β_1 & β) and repair rates (α and α_1) of the units. While their values decline with increase of abnormal weather rate (β) and failure rates (λ and λ_1) of the units. However, the effect of repair rate of the main unit is more. On the basis of the above results, it can be concluded that the system can be made more reliable to use by increasing repair rate of the both units.

IX. NOTATIONS

E	The set of regenerative states
MO/DO	Main/Duplicate unit is good and operative
$\overline{MWO}$ / $\overline{DWO}$	Main/Duplicate unit is good and waiting for operation in abnormal weather
MCs/DCs	Main/Duplicate unit is in cold standby mode
$\overline{MCs}$ / $\overline{DCs}$	Main/Duplicate unit is in cold standby mode in abnormal weather
λ/ λ_1	Constant failure rate of Main /Duplicate unit
β/ β_1	Constant rate of change of weather from normal to abnormal/abnormal to normal weather
MFur/DFur	Main/duplicate unit failed and under repair
MFUR/DFUR	Main/duplicate unit failed and under repair continuously from previous state
MFwr/DFwr	Main/duplicate unit failed and waiting for repair
MFWR/DFWR	Main/duplicate unit failed and waiting for repair continuously from previous state
$\overline{MFwr}$ / $\overline{DFwr}$	Main/Duplicate unit failed and waiting for repair due to abnormal weather
$\overline{MFWR}$ / $\overline{DFWR}$	Main/Duplicate unit failed and waiting for repair continuously from previous state due to abnormal weather
g(t)/G(t)	pdf/cdf of repair time of Main unit
$g_1(t)$/ $G_1(t)$	pdf/cdf of repair time of Duplicate unit
q_{ij} (t)/Q_{ij} (t)	pdf/cdf of passage time from regenerative state i to regenerative state j or to a failed state j without visiting any other regenerative state in (0,t]
$q_{ij.kr}$ (t)/ $Q_{ij.kr}$ (t)	pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k,r once in (0,t]

m_{ij}	The conditional mean sojourn time in regenerative state Si when system is to make transition in to regenerative state Sj. Mathematically, it can be written as, $$m_{ij} = E(T_{ij}) = \int_0^\infty td[Q_{ij}(t)] = -q_{ij}^{*\prime}(0)$$ where T_{ij} is the transition time from state Si to Sj; Si, Sj ε E.
μ_i	The mean Sojourn time in state Si this is given by $$\mu_i = E(T_i) = \int_0^\infty P(T_i > t)dt = \sum_j m_{ij}$$ where T_i is the sojourn time in state Si.
Ⓢ	Symbol for Laplace Stieltjes convolution
**	Symbol for Laplace Steiltjes Transform (L.S.T.)
´(desh)	Used to represent alternative result

X. REFERENCES

[1] Goel, L. R.; Sharma G. C. and Gupta, R. (1985). Cost analysis of a two-unit cold standby system under different weather conditions, Microelectronics Reliability, 25(4), p. 655-659.

[2] Gupta, R. and Goel, R. (1991). Profit analysis of a two-unit cold standby system with abnormal weather condition, Microelectronics Reliability, 31(1), p. 1-5.

[3] Kadian, M.S.; Chander, S. and Grewal, A. S. (2004). Stochastic analysis of nonidentical unit's reliability models with priority and different modes of failure, Decision and Mathematical Science, Vol. 9(1-3), p. 59 – 82.

[4] Malik, S.C; Deswal, S (2012): Stochastic Analysis of a Repairable System of Non-identical Units with Priority for Operation and Repair Subject to Weather Conditions. International Journal of Computer Applications (0975 – 8887) Volume 49– No.14, July 2012

[5] Kishan, Ram and Jain, Divya (2012). A two non-identical unit standby system model with repair, inspection and post-repair under classical and Bayesian viewpoints, Journal of reliability and statistical studies, 5(2), p. 85-103.

[6] Malik, S.C; Deswal, S and Sureria, J. K. (2013): Stochastic Analysis of a System of Non-identical Units with no Repair Activity in Abnormal Weather. Int. J. Agricult. Stat. Sci. (ISSN: 0973-1903), Vol. 9, Supplement 1, pp. 193-201, 2013

[7] Malik, S.C; Deswal, S (2015): Reliability Measures of a System of Two Non identical Units with Priority subject to Weather Conditions. Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666, Vol. 8, Issue 1 (2015): 181-190

[8] Kumar, J. & Goel, Meenu. (2016): Availability and profit analysis of a two-unit cold standby system for general distribution. Cogent Mathematics (2016), 3: 1262937

[9] Deswal, S. (2018): Effect of Weather on Priority Based Redundant System. Journal of Emerging Technologies and Innovative Research (JETIR - ISSN-2349-5162). March 2018, Volume 5, Issue 3