

Prediction of Noise Level by Mathematical Modeling in the Exhaust Muffler and Validation of these Analytical Results with the Experimental Results for 4-Stroke Diesel Engine

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Abstract

Exhaust noise in the internal combustion engine is known to be one of the most harmful pollutants to the mankind. However this noise can be reduced sufficiently by means of a well designed muffler. The present work aims the prediction of pressure drop by mathematical modeling in the exhaust muffler. The pressure drop in an exhaust muffler plays an important role for the design and development of a muffler. The prediction of pressure drop by mathematical modeling will be very useful for the design and development of muffler. The suitable design and development will help to reduce the noise level, but at the same time the performance of the engine should not be hampered by the back pressure caused by the muffler. This thought has implied the author to predict pressure drop a very important tool for the design of muffler. The author has solved one-dimensional wave equation by the method of separation variables using boundary conditions.

The author want to evaluate the pressure drop at the point x and at the time t along the muffler. Afterward to validate the theoretical results such as noise level, Engine performances, Brake thermal efficiency, Brake specific fuel consumptions with the existing muffler and the modified designed and fabricated muffler.

Keyword: Wave equation, Separable variable, Muffler.

1. Introduction

Internal combustion engines are typically equipped with an exhaust muffler to suppress the acoustic pulse generated by the combustion process. A high intensity

pressure wave generated by combustion in the engine cylinder propagates along the exhaust pipe and radiates from the exhaust pipe termination. The pulse repeats at the firing frequency of the engine which is defined by $f=(\text{engine rpm} \times \text{number of cylinders})/120$ for a four stroke engine. The frequency content of exhaust noise is dominated by a pulse at the firing frequency. In general, sound waves propagating along a pipe can be attenuated using either a resonative or a reactive muffler. Reactive and resonative silencers, which are commonly used in automotive applications, reflect the sound waves back towards the source and prevent sound from being transmitted along the pipe. Reactive silencer design is based either on the principle of a Helmholtz resonator or an expansion chamber, and requires the use of acoustic transmission line theory. In our experiment we have compared the experimental (with existing and modified muffler) and theoretical values.

2. Mathematical Modeling

The one dimensional wave equation is

$$\frac{\delta^2 u}{\delta t^2} = c^2 \frac{\delta^2 u}{\delta x^2} \dots\dots\dots (1)$$

Using boundary conditions

$$u = 0 \dots\dots\dots \text{at} \dots\dots\dots x = L \dots\dots \text{and} \dots \forall t \dots (2)$$

$$u = u_0 \dots\dots\dots \text{at} \dots\dots\dots t = 0, \dots\dots x = 0 \dots\dots (3)$$

$$\frac{\delta u}{\delta x} = 0 \dots\dots\dots \text{at} \dots\dots\dots x = 0 \dots\dots (4)$$

$$\frac{\delta u}{\delta t} = 0 \dots\dots\dots \text{at} \dots\dots t = 0 \dots\dots (0 \leq x \leq L) \dots (5)$$

The solution of can be written in the form

$$u(x, t) = X(x)T(t) \dots\dots\dots (6)$$

Substituting this values of u(x,t) in (1) we get

$$X'' - \mu X = 0 \dots\dots\dots (7)$$

$$T'' - c^2 \mu T = 0 \dots\dots\dots (8)$$

Using (2) we get $X(L) = 0$

Using (4) we get $X'(0) = 0$

For the solution of equation (7) three cases arises

Case-I

Let $\mu = 0$ then the solution of equation (7) is $X(x) = Ax + B$ and we get $A=0$ and $B=0$. So we reject $\mu = 0$

Case-II

let $\mu = \lambda^2 (\lambda \neq 0)$ Then the solution of equation (7) is

$$X(x) = Ae^{\lambda x} + Be^{-\lambda x}$$

we get $A=B=0$

So we again reject $\mu = \lambda^2 (\lambda \neq 0)$

Case-III

Let $\mu = -\lambda^2 (\lambda \neq 0)$

Then we get

$$X(x) = A \cos \lambda x + B \sin \lambda x$$

By using the conditions

$$X(L) = 0, X'(0) = 0$$

we obtain, $A \neq 0, B = 0$ and $L\lambda = \frac{1}{2}(2n-1)\pi, n = 1, 2, 3, \dots$

Hence non-zero solution of equation (7) becomes

$$X_n(x) = A_n \cos \frac{1}{2L}(2n-1)\pi x$$

Now equation (8) becomes $T'' + \frac{1}{4L^2}(2n-1)^2\pi^2 c^2 T = 0$

Whose solution is

$$T_n(t) = C_n \cos \frac{1}{2L}(2n-1)\pi ct + D_n \sin \frac{1}{2L}(2n-1)\pi ct$$

The general solution for the equation (1) becomes

$$u(x,t) = \sum_{n=1}^{\infty} [E_n \cos \frac{1}{2L}(2n-1)\pi ct + F_n \sin \frac{1}{2L}(2n-1)\pi ct] \cos \frac{1}{2L}(2n-1)\pi x$$

Applying boundary condition (3) we obtain $E_n = \frac{4u_0(-1)^{n-1}}{(2n-1)\pi}$

Again applying boundary condition (5) we obtain $F_n = 0$

Therefore the desired solution is

$$u(x,t) = \frac{4u_0}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos \frac{1}{2L}(2n-1)\pi ct \cos \frac{1}{2L}(2n-1)\pi x$$

$$u(x,t) = \frac{2u_0}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \left[\cos \frac{1}{2L} (2n-1)\pi(ct-x) \right.$$

i.e

$$\left. + \cos \frac{1}{2L} (2n-1)\pi(ct+x) \right]$$

Therefore $u(x,t) = u_1(ct-x) + u_2(ct+x)$

Where $u_1(ct-x) = \frac{2u_0}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos \frac{1}{2L} (2n-1)\pi(ct-x)$

$$u_2(ct+x) = \frac{2u_0}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos \frac{1}{2L} (2n-1)\pi(ct+x)$$

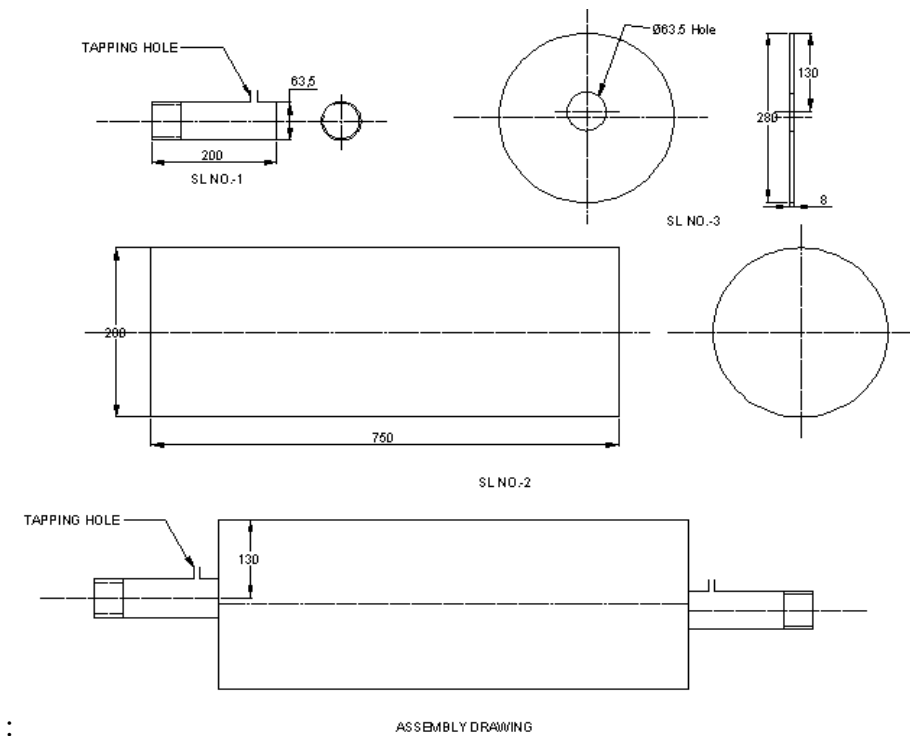
For positive going wave

$$u_2(ct+x) = 0$$

$$u(x,t) = u_1(ct-x) = \frac{2u_0}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$$

i.e $\cos \frac{1}{2L} (2n-1)\pi(ct-x)$

3. Design



Table

Sl. No.	Name	Quantity	Weight (Kg)	Thickness (mm)
1	Inlet pipe	1	0.5	2
2	Cylindrical drum	1	3.4	3
3	Side wall of drum	2	2.2	3
4	Exhaust pipe	1	0.5	2

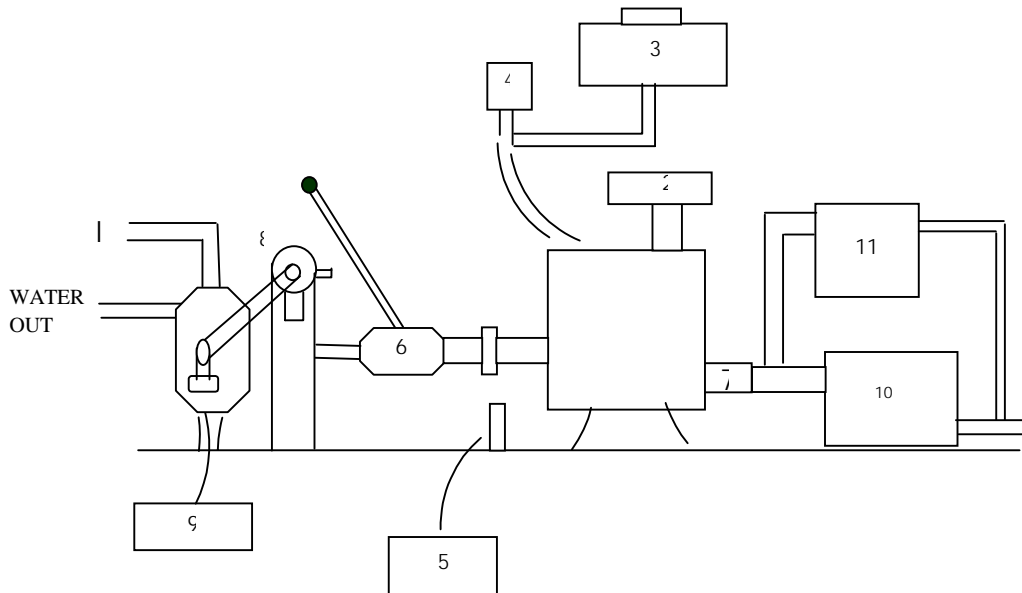


Figure : Experimental Setup

Experimental Setup

- 1.Engine
- 2.Air Filter
- 3.Diesel Tank
- 4.Diesel Measuring Burret
- 5.Speed Indicator
- 6.Clutch
- 7.Exhaust Pipe
- 8.Hydraulic Dynamometer
- 9.Torque Indicator
- 10.Reactive Muffler
- 11.Manometer

4. Methodology

The above schematic diagram shows the experimental set up, where we have taken reading of Fuel Consumption from diesel burette with the help of stop watch. At the same time with variation of loads we have measured the variations of Brake Horse Power with the help of dynamometer. The variation of sound level has been observed with the help of Sound level meter which has been placed one meter apart with an angle 45° with respect to exhaust end of the muffler.

5. Results and Discussion

Fig. 1 represents the sound level Vs BHP (KW). When sound level is 116.8db by without muffler, at that time we get sound level 96.4 db in existing muffler, 82.2 in our modified and fabricated muffler, 77.047 db by mathematical modeling. It is interesting to note down the variations between the sound levels % variation measured by modified and fabricated muffler & theoretical muffler is 4.3%.

Since the variation of sound levels by mathematical modeling & modified and fabricated muffler is of less order which justifies reasonable assumption taken in the mathematical modeling.

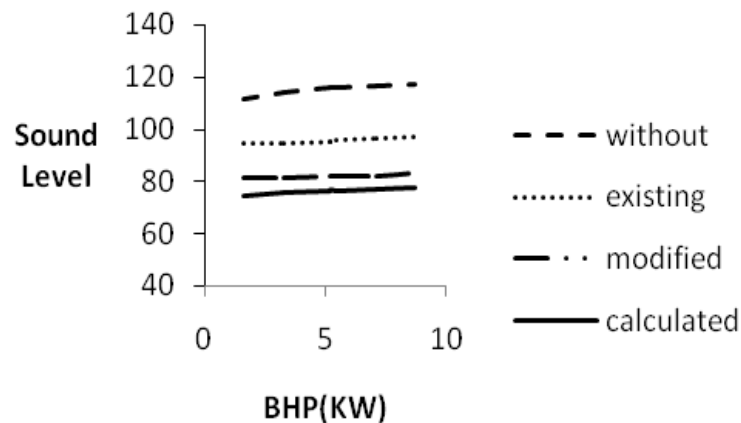


Figure 1: Comparison of sound Level among existing, modified and without muffler

Fig.2 represents the Comparison of Brake Thermal efficiency among existing, modified and without muffler. The maximum Brake Thermal efficiency without muffler, with existing muffler & with modified and fabricated muffler are 26.83, 25.44 & 25.11 respectively.

It is observed that Brake Thermal efficiencies without muffler and existing muffler are little higher than that of modified muffler.

The Brake Thermal efficiency with modified muffler is little less than the without muffler, because of the higher pressure drop in case of modified muffler in comparison to without and existing mufflers.

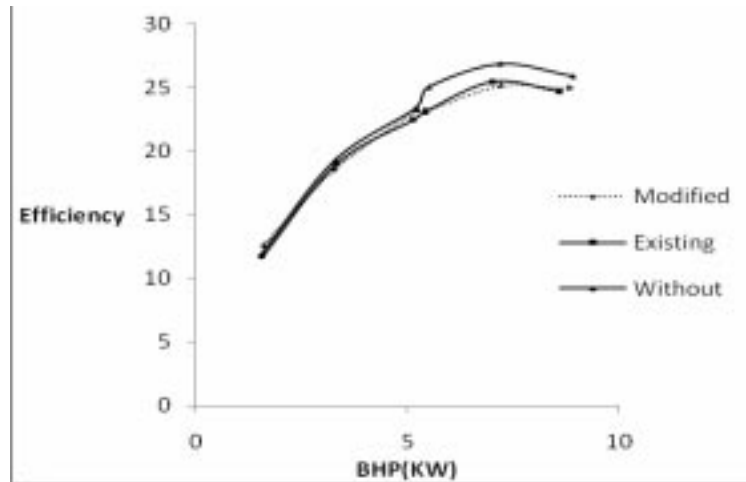


Figure 2: Comparison of efficiency among existing, modified and without muffler

Fig.3 represents the comparison of BSFC among existing, modified and without muffler. Here we can see that BSFC is 0.31 when it is without muffler. When we use existing muffler BSFC becomes 0.33. For modified and fabricated muffler BSFC is .33.

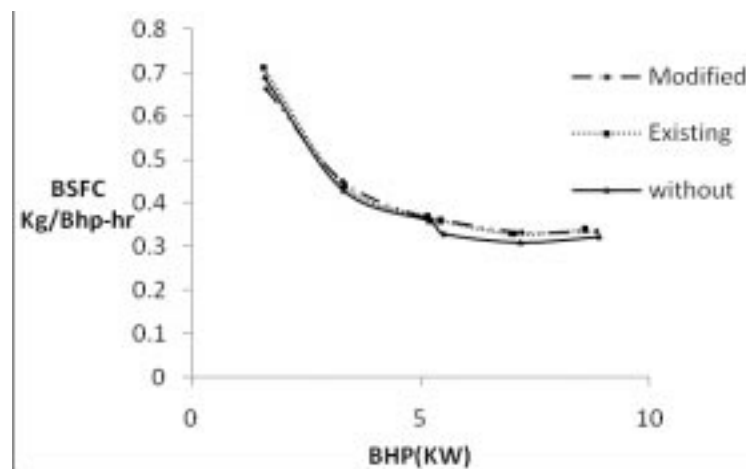


Figure 3: Comparison of BSFC among existing, modified and without muffler

6. Conclusion

From results and discussions the following conclusions are drawn:

1. Reduction of noise level is around 14db as compared to existing muffler.
2. The Brake Thermal Efficiency of engine is lower for modified and fabricated muffler as compared to existing muffler by 0.33%.
3. The Brake Specific Fuel Consumption is same with existing muffler.

7. Nomenclature

$u(x, t)$	Difference between the pressure at (x, t) and the outside pressure
u_0	Initial Sound pressure level (db)
c	Speed of sound (m / s)
L	Length of the muffler (m)
BSFC	Brake Specific Fuel Consumption
db	Decibel
f	Frequency
SPL	Sound Pressure Level
ω	Angular Velocity
t	Time (Second)

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