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Numerical Study of Radiative Unsteady MHD Casson Fluid with Heat Transfer in a Liquid Film Over a Stretching surface

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Abstract

In this present work, we examine the heat transfer of hydromagnetic boundary layer flow of unsteady Casson fluid over a stretching surface with thermal radiation. Similarity transformations are applied to convert the nonlinear partial differential equations to ordinary differential equations. To solve the system of non linear partial differential equations, Keller Box method is applied. The velocity and temperature profiles are examined graphically with variations of Casson parameter, Porosity parameter and Prandtl number. Results are good in agreement with the available literature.

Keywords: MHD, Casson Fluid, Liquid Film, Heat transfer.

1. INTRODUCTION

Many engineering problems, the analysis of flow and heat transfer of thin films has attracted much attention to many researchers because of their applications to industry such as cooling of metallic plates, polymers, fiber coating, rolling, annealing, and thinning of copper wires and processing of food etc.,. The quality of final product depends on the extrusion process which is very crucial and in the extrusion processes depends on the film flow and heat transfer characteristics of a thin liquid over a stretching plate.

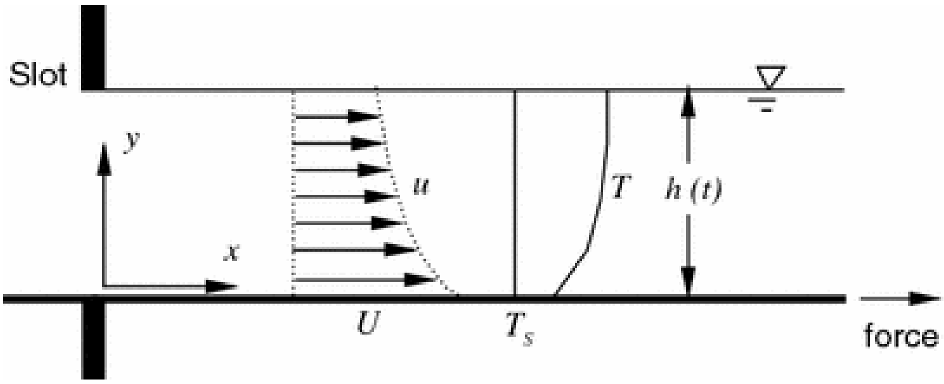
Applications of magnetohydrodynamics to metallurgy lies in the purification of molten metals from non- metallic inclusions by an application of magnetic field discussed by T.Y.Na,(1996). Hayat et al. (2008) discussed the effect of heat transfer and hall effect on the steady flow of a generalized Burgers fluid induced by a sudden pull of eccentric rotating disks. A.Ishak(2008) studied the local similarity solutions for laminar boundary layer flow along a moving cylinder in a parallel stream. In the presence of external magnetic field Subhas et al.(2009) studied heat transfer in a liquid film over an unsteady stretching surface with viscous dissipation. Megahed et al. (2012) studied the flow and heat transfer over an unsteady stretching sheet with internal heat generation and variable heat flux. Ibrahim et al.(2012) analyzed the unsteady MHD boundary-layer flow and heat transfer due to stretching sheet in the presence of heat source or sink. Rita et al.,(2013) used the theory of MHD in many applications such as extrusion of plastics in the manufacture of polymers, purification of crude oil etc.,. Jat et al.(2013) studied the steady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid over an exponentially stretching sheet in the presence of a uniform transverse magnetic field with radiative heat flux and viscous dissipation. Yan et al. (2015) carried out a comprehensive comparison on vibration and heat transfer of two elastic heat transfer tube bundles. Boltenko et al. (2015) investigated both heat transfer and pressure drop in an annular channel with heat transfer intensifiers. Megahed et al., (2015) investigated MHD viscous Casson fluid flow and heat transfer of second order slip velocity and thermal slip over a permeable stretching sheet in the presence of thermal radiation and internal heat generation. Baag et al. (2016) discussed the entropy generation for visco-elastic MHD fluid flow over a stretching sheet embedded in a porous medium. The effects of magnetic field and permeability of the medium on the flow field have been investigated. Yiping et al. (2016) studied numerically the effect of heat transfer enhancement by dimpled surface heat exchanger in thermoelectric generator. Zeeshan et al. (2016a) discussed the effect of magnetic dipole and heat transfer analysis on Jeffery fluid flow over a stretching sheet with suction or injection. Also, Zeeshan et al. (2016b) studied the effect of magnetic dipole on viscous ferro-fluid past a stretching surface in presence thermal radiation. Sridhar et al.(2016) studied the numerical solution to mass transfer on MHD flow of Casson fluid with suction and chemical reaction using Keller Box method. Karmina et al. (2017) examined MHD boundary layer flow of Casson fluid with heat transfer in a liquid film over unsteady stretching plate. A.Jamaludin et al.(2017) studied the boundary layer flow and heat transfer in a viscous fluid over a stretching sheet with viscous dissipation, internal heat generation and prescribed heat flux. M.G.Reddy et al.(2017) studied numerically the mass transfer flow of MHD radiative tangent hyperbolic fluid over a cylinder.

Recently, Sridhar et al.(2019) discussed the numerical study to diffusion of chemically reactive species over MHD exponentially stretching surface of a Casson fluid.

In view of the above discussion, the main objective of this study is to apply Keller Box method to find the approximate solution of nonlinear differential equations governing the fluid of flow and heat transfer of the current problem.

2. MATHEMATICAL FORMULATION

The MHD boundary layer flow over a flat plate is governed by the continuity and the Navier-Stokes equations for an incompressible viscous fluid. The fluid is electrically conducting under the influence of an applied magnetic field (x) normal to the stretching sheet. The induced magnetic field is neglected. A thin liquid film of uniform thickness h(t) lies on the horizontal surface. The surface heat flux q(x,t) at the stretching sheet varies with the power of distance x from the slit and with the inverse power of time factor t. The flow model is shown below in figure.



The resulting boundary-layer equations for such type of flow are (Eqs. 1-3):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\nu}{k}u-\sigma\frac{B^2(x)}{\rho}u \tag{2}$$

$$\rho C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y} \tag{3}$$

where, u and v are the velocity components in the x and y directions, respectively. Also v, ρ and σ are the kinematic viscosity, density and electrical conductivity of the fluid. Parameter of the Casson fluid is $\beta=\frac{\mu_B\sqrt{2\pi}}{P}$, T is the temperature and k is the thermal diffusivity coefficient of the fluid and a transverse magnetic field of uniform strength $B(x)$ is equal to $B(x) = B_0 x^{n-\frac{1}{2}}$,q_r is radiative heat flux and c_p is the specific heat.

Using Rosseland approximation for radiation (Brewster[18]),writing

$$q_r=-\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y} \tag{4}$$

Where σ is the Stefan-Boltzman constant and k^* is the absorption coefficient.

Assuming that the temperature difference within the flow is such that T^4 may be expanded in a Taylor series and expanding T^4 about T_∞ and neglecting the higher order terms, we have

$$T^4 \equiv 4T_\infty^3 T - 3T_\infty^4 \quad (5)$$

The boundary conditions are given below in Eqs. 4 and 5:

$$t = 0, u(t,x,y) = ax, v(t,x,y) = 0$$

$$T = T_w(x,t) \text{ at } y = 0$$

$$u(t,x,y) = v(t,x,y) = T(t,x,y) = 0 \text{ at } y > 0$$

$$t > 0 : u = ax, v = 0, T = T_w(x,t) \text{ at } y = 0 \quad (6)$$

$$u = 0, T = T_\infty \text{ as } y \to \infty \quad (7)$$

Here $T_w = T_\infty + \dfrac{c\xi^{\frac{-3}{2}} x^2}{2\nu}$ and T_0 is a heating or cooling temperatures. To solve the above problem, momentum and energy equations are non-dimensionalized first by introducing the following dimensionless variables.

Let us define the stream function ψ where $u = \dfrac{\partial \psi}{\partial y}$ and $v = -\dfrac{\partial \psi}{\partial x}$

Applying the following transformation

$$\psi = \sqrt{a\nu\xi}\, x f(\eta,\xi); \quad \eta = \sqrt{\frac{a}{\nu\xi}}\, y; \quad \xi = 1 - e^{-\tau}; \quad \tau = at;$$

$$T = \left[T_\infty + \frac{c\xi^{-\frac{3}{2}} x^2}{2\nu} \right] \theta(\eta); \quad (8)$$

$M = \dfrac{\sigma B^2(x)}{a\rho}, \lambda = \dfrac{\nu}{aK'}$ where λ= porosity parameter

Substituting (8) into equations (1)-(3)

$$\left(1 + \frac{1}{\beta}\right)\frac{\partial^3 f}{\partial \eta^3} + \frac{1}{2}(1-\xi)\eta \frac{\partial^2 f}{\partial \eta^2} + \xi\left(f\frac{\partial^2 f}{\partial \eta^2} - \left(\frac{\partial f}{\partial \eta}\right)^2 - M\frac{\partial f}{\partial \eta} - \lambda \frac{\partial f}{\partial \eta}\right) = \xi(1-\xi)\frac{\partial^2 f}{\partial \eta \partial \xi} \quad (9)$$

$$\frac{1}{2}\frac{\partial\theta}{\partial\eta}\eta(\xi-1)+\frac{3}{2}\theta\left(\xi^3-\xi^2\right)+2\xi\frac{\partial f}{\partial\eta}\theta-\xi f\frac{\partial\theta}{\partial\eta}=\frac{1}{\Pr}\left(1+\frac{4N}{3}\right)\frac{\partial^2 f}{\partial\eta^2} \tag{10}$$

where

$\Pr=\frac{\nu}{K^*}$ and $N=\frac{4\sigma T_\infty^3}{kk^*}$

The boundary conditions (4) and (5) becomes

$$f(0,\xi)=0,\ \frac{\partial \mathrm{f}}{\partial\eta}(0,\xi)=1,\ \theta(0,\xi)=1 \text{ at } \eta=0 \tag{11}$$

$$\frac{\partial \mathrm{f}}{\partial\eta}(\infty,\xi)\to 0,\ \theta(\infty,\xi)\to 0 \text{ as } \eta\to\infty \tag{12}$$

Considering $\xi=1$, equations (9) and (10) are transformed into

$$\left(1+\frac{1}{\beta}\right)f'''+ff''-f'^2-(M+\lambda)f'=0 \tag{13}$$

$$\frac{1}{Pr}\left(1+\frac{4N}{3}\right)\theta''-2f'\theta+f\theta'=0 \tag{14}$$

The boundary conditions becomes

$$at\,\eta=0,\ f(\eta)=0,\ f'(\eta)=1,\ \theta(\eta)=1 \tag{15}$$

$$as\,\eta\to\infty,\ f'(\eta)\to 0,\ \theta(\eta)\to 0 \tag{16}$$

Numerical Solution:

Equations (13)-(14) subject to boundary conditions(15)-(16) are solved by keller box method, as described by cebeci et al.(1988). The steps followed are

1. Reduce the equations (13)-(14) to a first order equation
2. Using central differences, write the difference equations
3. The resulting algebraic equation are linearized by Newton's method and writing them in matrix vector form
4. To solve the linear system,the block tri-diagonal elimination technique is employed

Introduce

$f'=p,$ (17)

$p'=q,$ (18)

$g'=t\ (g=\phi)$ (19)

equations (13)-(14) reduces to

$$\left(1+\frac{1}{\beta}\right)q' + fq - p^2 - (M+\lambda)p = 0 \quad (20)$$

$$t' + \frac{3Pr}{(3+4N)}(ft - 2pg) = 0 \quad (21)$$

consider the segment η_{j-1}, η_j with $\eta_{j-1/2}$ as the mid-point $\eta_0=0, \eta_j = \eta_{j-1}+h_j, \eta_J= \eta_\infty$ (22)

where h_j is the $\Delta\eta$ spaces and j=1,2,……..J is a sequence of number that indicates the coordinate locations.

Introducing finite differences for equations (17)-(21)

$$\frac{f_j - f_{j-1}}{h_j} = \frac{p_j + p_{j-1}}{2} = p_{j-1/2} \quad (23)$$

$$\frac{p_j - p_{j-1}}{h_j} = \frac{q_j + q_{j-1}}{2} = q_{j-1/2} \quad (24)$$

$$\frac{g_j - g_{j-1}}{h_j} = \frac{t_j + t_{j-1}}{2} = t_{j-1/2} \quad (25)$$

$$\left(1+\frac{1}{\beta}\right)\frac{q_j - q_{j-1}}{h_j} + \left(\frac{f_j + f_{j-1}}{2}\right)\left(\frac{q_j + q_{j-1}}{2}\right) - \left(\frac{p_j + p_{j-1}}{2}\right)^2 - (M+\lambda)\left(\frac{p_j + p_{j-1}}{2}\right) = 0 \quad (26)$$

$$\frac{t_j - t_{j-1}}{h_j} + \left(\frac{3\Pr}{3+4N}\right)\left(\left(\frac{f_j + f_{j-1}}{2}\right)\left(\frac{t_j + t_{j-1}}{2}\right) - .2\left(\frac{p_j + p_{j-1}}{2}\right)\left(\frac{g_j + g_{j-1}}{2}\right)\right) = 0 \quad (27)$$

Newton's method

Linearizing the non linear system of equations (23) to (27)

Introduce

$$f_j^{(k+1)} = f_j^{(k)} + \delta f_j^{(k)} \quad (28)$$

$$p_j^{(k+1)} = p_j^{(k)} + \delta p_j^{(k)} \quad (29)$$

$$q_j^{(k+1)} = q_j^{(k)} + \delta q_j^{(k)} \quad (30)$$

$$g_j^{(k+1)} = g_j^{(k)} + \delta g_j^{(k)} \quad (31)$$

$$t_j^{(k+1)} = t_j^{(k)} + \delta t_j^{(k)} \quad (32)$$

Substitute in equations (23) to (27)

Write

$$\delta f_j - \delta f_{j-1} - \frac{h_j}{2}\left(\delta p_j + \delta p_{j-1}\right) = (r_1)_{j-\frac{1}{2}} \tag{33}$$

$$\delta p_j - \delta p_{j-1} - \frac{h_j}{2}\left(\delta q_j + \delta q_{j-1}\right) = (r_2)_{j-\frac{1}{2}} \tag{34}$$

$$\delta g_j - \delta g_{j-1} - \frac{h_j}{2}\left(\delta t_j + \delta t_{j-1}\right) = (r_3)_{j-\frac{1}{2}} \tag{35}$$

$$(a_1)_j \delta q_j + (a_2)_j \delta q_{j-1} + (a_3)_j \delta f_j + (a_4)_j \delta f_{j-1} + (a_5)_j \delta p_j + (a_6)_j \delta p_{j-1} = (r_4)_{j-\frac{1}{2}} \tag{36}$$

$$(b_1)_j \delta t_j + (b_2)_j \delta t_{j-1} + (b_3)_j \delta f_j + (b_4)_j \delta f_{j-1} + (b_5)_j \delta p_j + (b_6)_j \delta p_{j-1} + (b_7)_j \delta g_j + (b_8)_j \delta g_{j-1} = (r_5)_{j-\frac{1}{2}} \tag{37}$$

where,

$$(a_1)_j = 1 + \frac{\beta h_j}{4(\beta+1)}(f_j + f_{j-1}) \tag{38}$$

$$(a_2)_j = (a_1)_j - 2.0 \tag{39}$$

$$(a_3)_j = \frac{\beta h_j}{4(\beta+1)}(q_j + q_{j-1}) \tag{40}$$

$$(a_4)_j = (a_3)_j \tag{41}$$

$$(a_5)_j = -\frac{\beta h_j}{2(\beta+1)}\left((p_j + p_{j-1}) + (M+\lambda)\right) \tag{42}$$

$$(a_6)_j = (a_5)_j \tag{43}$$

$$(b_1)_j = 1 + \frac{3\Pr h_j}{4(4N+3)}(f_j + f_{j-1}) \tag{44}$$

$$(b_2)_j = (b_1)_j - 2.0 \tag{45}$$

$$(b_3)_j = \frac{3\Pr h_j}{4(4N+3)}(t_j + t_{j-1}) \tag{46}$$

$$(b_4)_j = (b_3)_j \tag{47}$$

$$(b_5)_j = -\frac{3\Pr h_j}{2(4N+3)}(g_j + g_{j-1}) \tag{48}$$

$$(b_6)_j = (b_5)_j \tag{49}$$

$$(b_7)_j = -\frac{3\Pr h_j}{2(4N+3)}(p_j + p_{j-1}) \tag{50}$$

$$(b_8)_j = (b_7)_j \tag{51}$$

and

$$(r_1)_j = f_{j-1} - f_j + \frac{h_j}{2}\left(p_j + p_{j-1}\right) \tag{52}$$

$$(r_2)_j = p_{j-1} - p_j + \frac{h_j}{2}\left(q_j + q_{j-1}\right) \tag{53}$$

$$(r_3)_j = g_{j-1} - g_j + \frac{h_j}{2}\left(t_j + t_{j-1}\right) \tag{54}$$

$$(r_4)_j = q_{j-1} - q_j - \frac{\beta h_j}{4(\beta+1)}\left(f_j + f_{j-1}\right)\left(q_j + q_{j-1}\right) + \frac{\beta h_j}{4(\beta+1)}\left(p_j + p_{j-1}\right)^2 + \frac{(M+\lambda)\beta h_j}{2(\beta+1)}\left(p_j + p_{j-1}\right) \tag{55}$$

$$(r_5)_j = t_{j-1} - t_j - \frac{3\Pr h_j}{4(4N+3)}\left(f_j + f_{j-1}\right)\left(t_j + t_{j-1}\right) + \frac{3\Pr h_j}{2(4N+3)}\left(p_j + p_{j-1}\right)\left(g_j + g_{j-1}\right) \tag{56}$$

Taking j=1,2,3…

The system of equations becomes

$[A_1][\delta_1]+[C_1][\delta_2]=[r_1]$ (57)

$[B_2][\delta_1]+[A_2][\delta_2]+[C_2][\delta_3] = [r_2]$ (58)

….. $[B_{J-1}][\delta_{j-2}]+[A_{J-1}][\delta_{j-1}]+[C_{J-1}][\delta_j] = [r_{J-1}]$ (59)

$[B_J][\delta_{J-1}]+[A_J][\delta_J]= [r_J]$ (60)

where

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ d & 0 & 0 & d & 0 \\ 0 & d & 0 & 0 & d \\ (a_2)_1 & 0 & (a_3)_1 & (a_1)_1 & 0 \\ 0 & (b_2)_1 & (b_3)_1 & 0 & (b_1)_1 \end{bmatrix} \tag{61}$$

$$A_j = \begin{bmatrix} d & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & d & 0 \\ 0 & -1 & 0 & 0 & d \\ (a_6)_j & 0 & (a_3)_j & (a_1)_j & 0 \\ (b_6)_j & (b_8)_j & (b_3)_j & 0 & (b_1)_j \end{bmatrix} \quad (62)$$

$$B_j = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & d & 0 \\ 0 & 0 & 0 & 0 & d \\ 0 & 0 & (a_4)_j & (a_2)_j & 0 \\ 0 & 0 & (b_4)_j & 0 & (b_2)_j \end{bmatrix} \quad (63)$$

$$C_j = \begin{bmatrix} d & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ (a_5)_j & 0 & 0 & 0 & 0 \\ (b_5)_j & (b_7)_j & 0 & 0 & 0 \end{bmatrix} \quad (64)$$

The Block Elimination Method

The block tri-diagonal structure of the linearized differential equations of the system can be written as

$$\begin{bmatrix} [A_1] & [C_1] & & & \\ [B_2] & [A_2] & [C_2] & & \\ & & \vdots & & \\ & & [B_{J-1}] & [A_{J-1}] & [C_{J-1}] \\ & & & [B_J] & [A_J] \end{bmatrix} \begin{bmatrix} [\delta_1] \\ [\delta_2] \\ \vdots \\ [\delta_{J-1}] \\ [\delta_J] \end{bmatrix} = \begin{bmatrix} [r_1] \\ [r_2] \\ \vdots \\ [r_{J-1}] \\ [r_J] \end{bmatrix} \quad (65)$$

This is of the form $A\,\delta = r$ (66)

To solve the above system

Write [A] = [L] [U] (67)

where

$$L = \begin{bmatrix} [\alpha_1] & & & \\ [\beta_2] & [\alpha_2] & & \\ & & \ddots & \\ & & [\alpha_{j-1}] & \\ & & [\beta_J] & [\alpha_J] \end{bmatrix} \qquad (68)$$

and

$$U = \begin{bmatrix} [I] & [\Gamma_1] & & & \\ & [I_2] & [\Gamma_2] & & \\ & & \ddots & & \\ & & & [I] & [\Gamma_{J-1}] \\ & & & & [I] \end{bmatrix} \qquad (69)$$

Where [I] is the identity matrix

$[\alpha_i][\Gamma_i]$ are determined by the following equations

$[\alpha_1] = [A_1]$

$[A_1][\Gamma_1] = [C_1]$

$[\alpha_j] = [A_j]-[B_j][\Gamma_{j-1}]$ j=2,3,. ....,J

$[\alpha_j][\Gamma_j] = [C_j]$ j=2,3,..........,J-1

Substituting (67) in (66)

$LU\delta = r$

Let $U\delta = W$

then $LW = r$

where $W = \begin{bmatrix} [w_1] \\ [w_2] \\ \vdots \\ [w_{j-1}] \\ [w_J] \end{bmatrix}$

Now $[\alpha_1]\,[w_1] = [r_1]$

$[\alpha_j]\,[w_J] = [r_J]-[B_J][W_{j-1}]$ for $2 \leq j \leq J$

once the elements of W are found,

substitute in $U\delta=W$

and solve for δ

$[\delta_J] = [W_J]$

$[\delta_J] = [W_J]-[\Gamma_J][\delta_{J+1}]$, $1 \leq j \leq J-1$

These calculations are repeated until some convergence criterion is satisfied and we stop the calculations when $|\delta g_0^{(i)}| \leq \varepsilon$, where ε is very small prescribed value taken to be $\varepsilon = 0.00001$.

5. RESULTS AND DISCUSSION

Numerical computations are carried out for different physical parameters, such as Casson parameter β, Hartmann number M, porosity parameter λ, Radiation parameter N and Prandtl number Pr to validate the results. we observed that from Figs. 1-3 the magnitude of velocity and boundary layer thickness decreases with an increase in fluid parameter β, M and λ. The momentum boundary layer thickness decreases with increase in Casson parameter β is observed. It is observed from the Fig. 4 that the effect of radiation is not much effective on flow. From Figs. 5-7, shows that the temperature increases with increase in Casson parameter, radiation parameter and decreases with increase in Prandtl number. An increase in Prandtl number, radiation reduces the thermal boundary layer thickness.

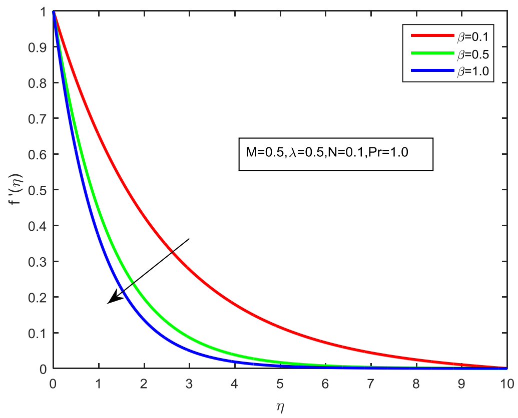


Fig. 1: Variation of velocity with Casson Parameter β

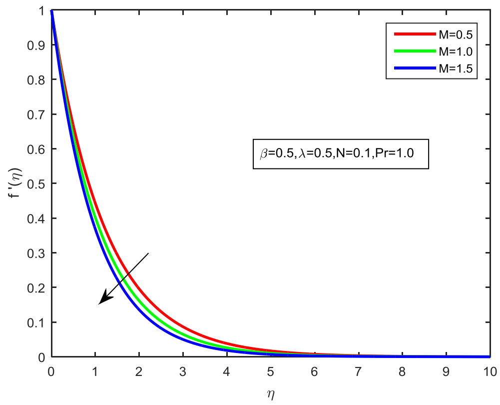


Fig. 2: Variation of velocity with Magnetic Parameter β

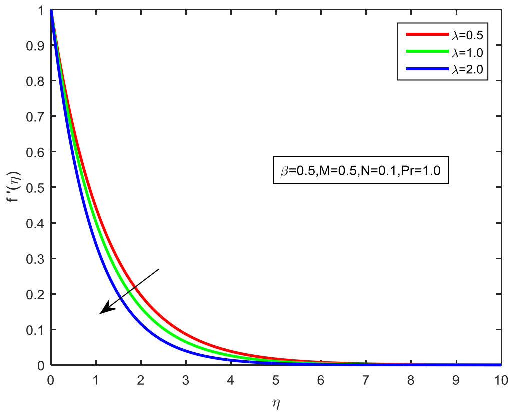


Fig. 3: Variation of velocity with Porous Parameter λ

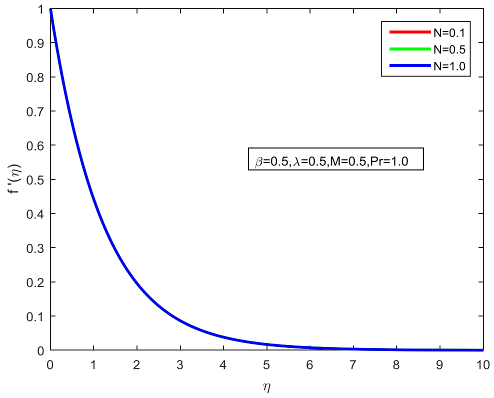


Fig. 4: Variation of velocity with Radiation ParameterN

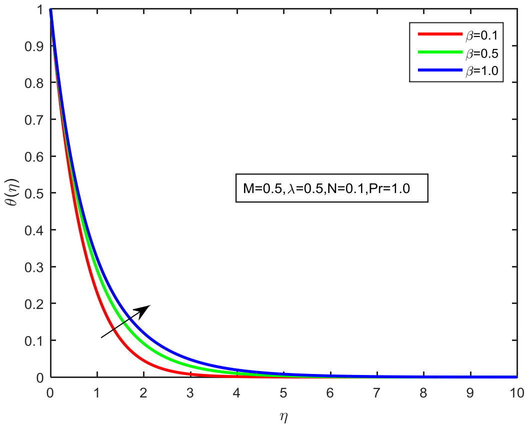


Fig. 5: Variation of temperature with Casson Parameter β

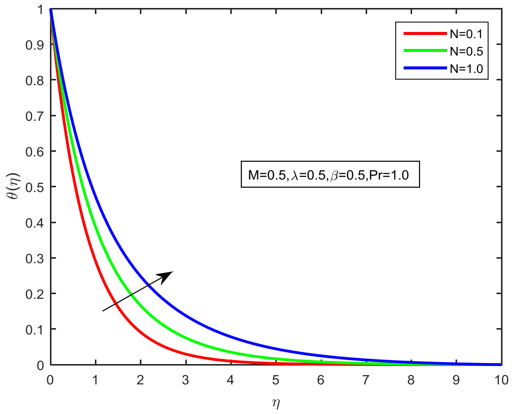


Fig. 6: Variation of temperature with Radiation N

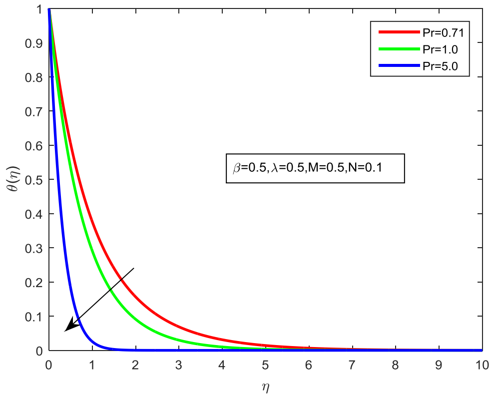


Fig. 7: Variation of temperature with Prandtl number Pr

6. CONCLUSIONS

MHD boundary layer flow of radiative unsteady Casson fluid with heat transfer over a stretching surface is examined in this work.

The notable marks of this investigation are

1. Velocity is found to be decreased with increasing Casson parameter β, magnetic parameter M, Porous parameter λ.
2. Radiation does not affect the flow.
3. The temperature is found to be increasing with increase in Casson parameter β, radiation N.
4. The temperature is found to be decreasing with increasing in Prandtl number Pr.

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