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Magma into Commutative Magma

U. Ananda Reddy[1], Dr. G. Shobhalatha[2] and Dr. L. Sreenivasulu Reddy[3]

*[1]Reseach Scholar, Department of Mathematics, S K University,
Ananthapurum, Andhra Pradesh, India.
[2]Professor, Department of Mathematics, S K University,
Ananthapurum, Andhra Pradesh, India.
[3] Academic Consultant, Department of Mathematics, S V University,
Tiruapti, Andhra Pradesh, India.
E-mails: anandareddy5244@gmail.com, gshobhalatha@yahoo.com,
sreenivasulureddy.svu@gmail.com*

Abstract

A mapping $*: X \times X \rightarrow X$ is a (binary) operation, and the pair $(X,*)$ is named as a Magma [1] . Magma with the property: $x*(y*z) = z*(x*y) = y*(z*x)$ for all x, y, z in Magma is named as L-cyclic magma or with the property $(x*y)*z = (z*x)*y = (y*z)*x$ for all x, y, z in Magma is named as R-cyclic magma. In this paper, every result show only identical goal, that is , to prove when a L-cyclic magma or R-cyclic magma becomes commutative magma .

***Keywords*:** Magma, L-cyclic magma, R-cyclic magma, L-R-cyclic magma, commutative magma.

1. Introduction

The algebraic objects encountered in this chapter are sets with a binary operation defined on them .Andreas[1] introduced a term “magma” in his Ph.D., theses with entitle “ Classification and Enumeration of finite semigroups” . Magma nothing but an algebraic structure with one binary operation on a nonempty set. Throughout this paper , we consider the magma with atleast any one of the property

i. $x*(y*z) = z*(x*y) = y*(z*x)$ for all x, y, z in Magma
   or
ii. $(x*y)*z = (z*x)*y = (y*z)*x$ for all x, y, z in Magma

The magma with first property is named as L-cyclic magma ,with second property is named as R-cyclic magma. If it has both properties ,then it is named as L-R-cyclic magma.

Throughout this paper , cross cancellation property in a magma means for any x, y and z in a magma , either $x * y = z * x \Rightarrow y = z$ or $y * x = x * z \Rightarrow y = z$. An element e in a magma is idempotent if $e * e = e$.

L-identity element means left identity element .

R-identity element means right identity element .

Magma with commutative is named as commutative magma.

This paper contains two sections :Section 1: it contains the introduction and overview of the paper and section 2 :It shows some of the results when a L-cyclic magma or R-cyclic magma into commutative magma and finally follows references .

2. L-cyclic *magma or R-cyclic magma into commutative magma:*

In this section ,all results shows only one goal that is when a L-cyclic magma or R-cyclic magma becomes commutative one. each L-cyclic magma or R-cyclic magma consider with one of the additional property:- idempotent, L- identity or R-identity.

Result 2.1: A L-cyclic Magma $(S,*)$ with idempotent element e is commutative magma if it has a cross cancellation property.

Proof:

If e be an idempotent element of magma $(S,*)$, then $e * e = e$

Thus $x * (e * e) = x * e$,for any x in S.

since S has a L- cyclic property ,so $x * (e * e) = e * (x * e)$

Therefore, $e * (x * e) = x * e$

By using cross cancelation property $e * (x * e) = x * e$,we have $x * e = x$.

Since x is an arbitrary element in , so the idempotent element e is a right identity in magma $(S,*)$.

Thus every idempotent element e in magma $(S,*)$ is a right identity

For proving left identity next consider and element $e * (e * x)$

By using L-cyclic property on $e * (e * x)$, we have $e * (e * x) = x * (e * e)$

Since e is idempotent element ,therefore $x * (e * e) = x * e$

Thus, $e * (e * x) = x * e$

Applying cross cancelation property on $e * (e * x) = x * e$, we have $e * x = x$, $\forall x$ in .

Since x is an arbitrary element in , so the idempotent element e is a left identity in magma $(S,*)$.

Thus every idempotent element e in magma $(S,*)$ is a left identity

Hence, the idempotent element e is identity in $(S,*)$.

Finally to show that commutative property of $(S,*)$.

Since e is the identity of $(S,*)$,so for any x, y in S , we have $x * y = (e * x) * (e * y)$. Since S has L-cyclic property, so $(e * x) * (e * y) = y * \big((e * x) * e\big)$ and

$y * \big((e * x) * e\big) = e * \big(y * (e * x)\big)$.

Since e is the identity , so $e * \big(y * (e * x)\big) = e * (y * x)$ and $e * (y * x) = y * x$.

Thus $x * y = y * x$ for any x, y in S.

Result 2.2: A R-cyclic Magma $(S,*)$ with idempotent element e is commutative magma if it has a cross cancellation property.

Proof:

If e be an idempotent element of magma $(S,*)$, then $e * e = e$

Thus, $(e * e) * x = e * x$ for all x in .

since S has a R-cyclic property ,so $(e * e) * x = (x * e) * e$ and $(x * e) * e = (e * x) * e$.

Therefore, $(e * x) * e = e * x. \Rightarrow (e * x) * e = e * x$, for all x in S .

By using cross cancelation property on $(e * x) * e = e * x$,so we have $e * x = x$.

Since x is an arbitrary element in , so the idempotent element e is a left identity in magma $(S,*)$. Thus every idempotent element e in magma $(S,*)$ is a left identity

For proving right identity next consider and element $(e * x) * e$.

By using R-cyclic property on $(e * x) * e$, we have $(e * x) * e = (e * e) * x$ and $(e * e) * x = (x * e) * e$.

Since e is left identity in S , we have $(e * e) * x = e * x$ & $e * x = x$

Therefore, $(x * e) * e = e * x$ ..

By using cross cancellation property on $(x * e) * e = e * x$, we have $x * e = x$

Thus , $x * e = x$

Since x is an arbitrary element in , so the idempotent element e is a right identity in magma $(S,*)$.

Thus every idempotent element e in magma $(S,*)$ is a right identity

Hence, the idempotent element e is identity in $(S,*)$.

Finally to show that commutative property of $(S,*)$.

Since e is the identity of $(S,*)$,so for any x, y in S , we have $x * y = (e * x) * (e * y)$.

Since S has R-cyclic property, so $(e * x) * (e * y) = \big((e * y) * e\big) * x$ and $\big((e * y) * e\big) * x = \big((e * e) * y\big) * x$.

Since e is the identity of $(S,*)$, so we have $\big((e * e) * y\big) * x = (e * y) * x$ and $(e * y) * x = y * x$.

Thus $x * y = y * x$ for any x, y in S.

Result 2.3: A L-cyclic Magma $(S,*)$ with L-identity element e is commutative magma.

Proof:

Let $x, y\ z$ be an arbitrary elements in magma S .

By using L-cyclic property on $* (y * z)$, we have $x * (y * z) = z * (x * y)$.

Again using L-cyclic property on $* (x * y)$, we have $z * (x * y) = y * (z * x)$

Since e is the left identity of $(S,*)$,so for any x, y in S , we have $x * y = (e * x) * (e * y)$. Since S has L-cyclic property, so $(e * x) * (e * y) = y * \big((e * x) * e\big)$ and

$y * \big((e * x) * e\big) = e * \big(y * (e * x)\big)$.

Since e is the left identity , so $e * \big(y * (e * x)\big) = e * (y * x)$ and $e * (y * x) = y * x$.

Thus $x * y = y * x$ for any x, y in S.

Result 2.4: A R-cyclic Magma $(S,*)$ with L-identity element e is commutative magma.

Proof:

Let $x, y\ z$ be an arbitrary elements in magma S .

By using R-cyclic property on $(x * y) * z$,we have $(x * y) * z = (z * x) * y$.

Again using R-cyclic property on $(z * x) * y$,we have $(z * x) * y = (y * z) * x$.

Since e is the left identity on $(S,*)$, so for any x, y in S ,we have $x * y = (e * x) * (e * y)$.

Since S has R-cyclic property, so $(e * x) * (e * y) = \big((e * y) * e\big) * x$, $\big((e * y) * e\big) * x = (y * e) * x$ and $(y * e) * x = (y * x) * e$.

Again using R-cyclic property on $(y * x) * e$, we have $(y * x) * e = (e * y) * x$.

Since e is the left identity, so $(e * y) * x = y * x$.

Thus $x * y = y * x$ for any x, y in S.

Result 2.5: A L-cyclic Magma $(S,*)$ with R-identity element e is commutative magma.

Proof:

Let $x, y \ z$ be an arbitrary elements in magma S .

By using L-cyclic property on $* \ (y * z)$, we have $x * (y * z) = z * (x * y)$.

Again using L-cyclic property on $* \ (x * y)$, we have $z * (x * y) = y * (z * x)$

Since e is the right identity on $(S,*)$, so for any x, y in S ,we have $x * y = (x * e) * (y * e)$. Since S has L-cyclic property, so $(x * e) * (y * e) = e * \big((x * e) * y\big)$, $e * \big((x * e) * y\big) = y * \big(e * (x * e)\big)$, $y * \big(e * (x * e)\big) = y * \big(e * (e * x)\big)$ and $y * \big(e * (e * x)\big) = y * \big(x * (e * e)\big)$.

Since e is the right identity, so $y * \big(x * (e * e)\big) = y * (x * e)$ and $y * (x * e) = y * x$.

Thus $x * y = y * x$ for any x, y in S.

Result 2.6: A R-cyclic Magma $(S,*)$ with R-identity element e is commutative magma.

Proof:

Let $x, y \ z$ be an arbitrary elements in magma S .

By using R-cyclic property on $(x * y) * z$,we have $(x * y) * z = (z * x) * y$.

Again using R-cyclic property on $(z * x) * y$ and it's result respectively ,we have $(z * x) * y = (y * z) * x = (x * y) * z$.

Since e is the right identity on $(S,*)$, so for any x, y in S ,we have $x * y = (x * e) * (y * e)$ Since S has R-cyclic property, so $(x * e) * (y * e) = \big((y * e) * x\big) * e$.

Since e is the right identity, so $\big((y * e) * x\big) * e = y * x$.

Thus $x * y = y * x$ for any x, y in S.

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